Intermediary Compensation under Endogenous Advice Quality in Insurance Markets

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Abstract:

We use Hotelling (1929)’s spatial competition approach to analyze insurance market outcomes when consumers face mismatch risk and can consult intermediaries to obtain product recommendations. Brokers can decide on both their level of advice quality and - if allowed by the regulator - on the size of kickbacks (rebates) offered to consumers, i.e., on what fraction of their commissions to pass on to their clients. We show that when kickbacks are banned, the market is characterized by a pooling equilibrium with either low or high advice quality. By contrast, when kickbacks are allowed, no separating equilibrium with both levels of advice quality exists, since the low-quality broker is always forced out of the market. Endogenous advice qualities promote a market equilibrium in which high advice quality and a direct distribution system coexist. Moreover, the market equilibrium with kickbacks results in higher consumer welfare than the equilibrium without kickbacks.

JEL Classification: G22, G38, L15, M52
Keywords: insurance regulation, insurance intermediation, remuneration schemes, advice quality.

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1 Introduction

Insurance markets differ considerably in terms of regulation. Since consumers usually vary in their preferences and needs, insurance companies offer policies with differing characteristics to match consumers’ preferences (Schlesinger and Von der Schulenburg 1991); however, consumer financial literacy (i.e., consumers’ qualification to assess the characteristics of insurance policies and their personal needs), particularly when it comes to insurance contracts, is limited, and so insurance brokers are the matchmakers in these markets.

Recent regulatory action in Europe and ongoing discussions in different countries worldwide evaluate the idea of minimum (advice) quality standards for insurance brokers and potential kickbacks to consumers. In the U.S., resale price maintenance restrictions prevent brokers from selling policies at lower prices than stated by the insurer (with commissions embedded in the retail price). This can be interpreted as "anti-rebating laws", which is a ban on brokers’ rebating any portion of their commission to the policyholder (Regan and Tennyson 2000). Hilliard et al. (2013) discuss the economic rationale for the choice and variety of distribution systems. Following these authors, "a common justification for these laws is to discourage agents from needlessly replacing policies as a way of increasing commission income" (p. 711). They further argue that resale price maintenance in the insurance industry can be justified for two reasons: (a) resale price restrictions on insurance products require agents to provide greater information services and (b) rebating may undermine customer persistency. A customer who will purchase only if offered a rebate has a lower valuation of the product, or of the services provided by the agent, than the customer who purchases at full price. If low-valuation customers are more likely to cash in their policies early, insurers may not recover the fixed costs of selling and underwriting these policies. Under this argument, insurers’ expectations of losing money on such customers could explain resale price restrictions" (p. 716).

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1 See Van Rossum (2005) for recent trends underlying the regulatory changes and the convergence between banking and insurance regulation. In response to the global financial crisis of 2008, modernization and improvement of insurance regulatory systems in the US and the European Union were demanded. Coope (2009), for example, examines the different approaches taken by the US and the EU in an effort to improve the effectiveness of state-based insurance regulation.

2 For simplicity, we use the term broker for all types of intermediaries.

3 Brokers do not only act as a distribution channel for insurance companies but also as market makers who match insurance needs of policyholders with insurers that have the capability of meeting those needs (Cummins and Doherty, 2006). In the U.S., insurance intermediaries are mostly compensated via commissions based on premium revenues sold (contingent commissions). Direct distribution (which includes online distribution in particular) is one of the growing distribution channels. As argued by Browne et al. (2014), contingent commission payments are beneficial to insureds because they provide an incentive for the broker to place their coverage with an insurer that is charging an adequate premium. Consumers generally suffer some kind of disutility when purchasing at an inadequate premium or getting an imperfect product. Therefore, a consumer’s willingness to pay for advice depends on both the advice quality of the broker and the individual mismatch risk.

The purpose of this paper is to analyze insurance market outcomes in a setting with minimum advice quality standards and kickbacks. In this way, the feature of our study is to evaluate different broker types coexisting in the market and the impact on consumer welfare. In fact, the possibilities of insurance brokers to differ in their advice quality and to pass commissions on to their clients enables them to engage in quality competition in addition to price competition. This new feature represents, in our view, a more realistic framework to evaluate an insurance market with intermediaries. This research provides economic arguments to evaluate the advantage of kickbacks and contributes to the ongoing discussion about intermediary compensation strategies.

Insurance intermediation has been researched before. For instance, a recent empirical study by Anagol et al. (2017) finds that life insurance agents offer poor advice quality and tend to confirm incorrect consumer beliefs. They also find speculative evidence that competition among agents can improve advice. Most theoretical studies focus on the simple case of a single broker when analyzing the impact of commissions on advice quality (Inderst and Ottaviani 2009; Inderst and Ottaviani 2012a) or they compare fee-based and commission-based remuneration systems when confronted with strategic advice (Inderst and Ottaviani 2012b). Gravelle (1993) uses a theoretical model to examine a commission system for the sale of complex life insurance products to passive consumers. He shows that when brokers are honest but give impartial advice the market equilibrium is inefficient. He builds on an earlier study with similar results, see Gravelle (1991). Focht et al. (2013) compare fee-based and commission-based compensation systems and find that these are payoff equivalent if the intermediary is completely honest. Hofmann and Nell (2011) show that, although insurers’ equilibrium profits are equivalent under both systems, social welfare is always higher under a fee-based than under a commission system. Schiller and Weinert (2018) study the case where advisors have an incentive to recommend unsuitable products to their consumers due to the existence of persuasion costs. Sonnenholzner et al. (2009) use a model in which a monopoly reinsurance broker can decide how much to invest in his advice quality in the presence of a potential new entrant. Seog (1999) explains the coexistence of two distribution systems, an exclusive agency system where each intermediary represents only one insurance company and an independent agency system where each intermediary represents more than one firm. Price dispersion and consumers’ search behavior are shown to affect the choice of the marketing system. Zweifel and Gherini (1990) also compare an exclusive agency system to an independent agency system.

Kickbacks (or rebates) are rarely the focus of economic models for intermediary compensation and additional research is needed. Crosby et al. (1991) as well as Regan and Tennyson (2000) study rebates. Crosby et al. (1991) emphasizes the effects of different rebating conditions in an experimental simulation of the life insurance sales situation; Regan and Tennyson (2000) analyze the choice of insurance distribution systems (with a focus on the US market) and the compensation structure is

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5 Another area of research studies why independent intermediaries have higher expense ratios and superior service as compared to exclusive agency firms (see for example Barrese and Nelson 1992, Barrese et al. 1995).
related to agent incentives to offer price discounts via commission rebating. In a subsequent paper \cite{Hilliard2013} focus on the distribution system choice, on issues regarding the insurer-agent relationship and structure of compensation, and on regulatory oversight of distribution.

Interestingly, when examining renumeration schemes, past research disregards the insurance company’s option of direct distribution. A reason might be the preceding structure of distribution channels: Direct distribution was less important in the past and limited to (mostly) standardized products. The IDD incorporates direct distribution emphasizing its growing importance. It is noteworthy that direct distribution systems are becoming progressively more important, especially in property and casualty insurance; e.g. in car insurance. However, in their insurance-distribution-survey 2014 the advisory, broking and solutions company Willis Towers Watson identifies direct distribution as one of the growing distribution channels with an increased market share of 11 percent. Likewise, \cite{Hilliard2013} point out that the direct distribution model becomes more appealing for many carriers due to the Internet and global call-center outsourcing.

This article contributes to and extends previous literature on insurance intermediation in several ways. First, incorporating a direct distribution system in a model of insurance intermediation provides a more realistic framework for an insurance market. Second, the model studies intermediaries who can decide (1) on their level of advice quality and (2), if allowed by regulation, on kickbacks to consumers. In contrast to previous literature, consumers have the option to purchase a policy directly or through one of the brokers. Results derived from this model aim to contribute to the ongoing discussion of who should pay for advisory services in the insurance industry. For the reason of analytic tractability, our model is static in nature and does not incorporate the possibility to lapse and renew insurance contracts. Consequently, it cannot speak to the common justifications for resale price maintenance as argued by \cite{Hilliard2013}. It does, however, bring forward a new argument against a ban of rebates: kickbacks may help improve the level of available advice quality levels and thereby increase consumer welfare.

The remainder of this article is structured as follows. Section 2 introduces the model. Section 3.1 discusses broker competition when kickbacks to consumers are prohibited. Section 3.2 analyzes outcomes when brokers can engage in both quality and price competition via kickbacks. Section 3.3 addresses consumer welfare under both kickback regimes. A last section concludes.

## 2 Model Framework

Our model is generally based on the common spatial competition model of product differentiation introduced by Hotelling (1929), which assumes that heterogeneous consumers have diverse preferences
across available brands or products. In what follows, we present in detail our assumptions regarding
the insurance marketplace, the broker marketplace, and the consumers.

2.1 Insurance Market

We study a Bertrand-competitive market with two risk-neutral insurance companies, labeled A and B, respectively. Each company sells a single insurance policy to consumers. These policies do not differ
in the level of insured risk but have different features or characteristics. Both firms are assumed to
have equal cost structures and, for simplicity and without loss of generality, produce at zero fixed costs
and constant marginal costs c.

The net price of the policies is \( p_n \in \{ p_A, p_B \} \). When selling the policies via a broker, the insurance
firms must pay the broker a commission \( \alpha p_n \) with \( \alpha \in (0, 1) \). Therefore, the gross price for a policy
is \( (1 + \alpha)p_n \). By contrast, in the case of direct distribution, there is no commission payment, and the
gross price equals the net price \( p_n \).

Each insurance company seeks to maximize its expected profit \( E(P_n) \). To this end, the net price \( p_n \) of
the firm’s policy is set such that

\[
\max_{p_n} E(P_n) = p_n - c.
\]

2.2 Broker Market

There are two risk-neutral brokers in the market, broker 1 and broker 2. Each broker offers advice
to consumers with a quality level \( q_i \), which represents the probability that a broker recommends the
right product. The level of a broker’s advice quality can be either high or low, denoted as \( q_H \) and \( q_L \),
respectively, with \( q_H > q_L \). The low advice quality level \( q_L \) can be interpreted as a minimum quality
standard required by the regulator, and implemented, e.g., via a certification process. Here, we assume
that \( q_L \in (\frac{1}{2}, 1) \), which guarantees that even low-quality broker advice is better than a coin toss. Each
broker can invest \( C_i > 0 \) in professional training to increase advice quality to \( q_H \in (q_L, 1] \) and obtain

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6 This model approach is contrasted by the representative consumer model often associated with Chamberlin et al. (1948) and further analyzed by Spence (1976, 1978), among others.
7 For brevity of notation, these labels also apply to the policies issued by the respective firm.
8 We assume the commission rate \( \alpha \) to be exogenous. It corresponds to the typical commission rate in the market.
9 This assumption is noncritical, because (1) policies are often calculated based on a fixed level of acquisition costs, and
(2) competition for brokers often implies a certain commission level. For example, when firms steer advice through
commissions - as discussed in Inderst and Ottaviani (2012a) - symmetry and consumers’ uniformly distributed
preferences lead to identical endogenous commissions.
10 In Europe, for instance, the new IDD introduced certain observable requirements for brokers that have to be met
in order to ensure the minimum quality standard imposed by regulation. For further information about the effects
of minimum quality standards in product differentiation models, see Boom (1995), Bockstael (1984), and, more

5
Consumers cannot directly observe their $\theta$. Instead, a consumer located at $\theta$ only observes a signal $s(\theta) = |\theta - \frac{1}{2}| \in [0, \frac{1}{2}]$, which describes that individual's exposure to mismatch risk.

Consumers receiving a signal $s(\theta) = 0$ know that they are not exposed to the risk of a potential mismatch. Hence, they are indifferent between the two policies and will purchase a policy directly, as soliciting further (costly) advice from a broker does not increase their utility. By contrast, consumers receiving a signal closer to $s(\theta) = \frac{1}{2}$ know that one of the policies is (much) more suitable than the other, however, they don’t know which policy is the better one. Hence, these consumers are highly exposed to mismatch risk, which can make asking a broker for additional advice attractive.

Following Hotelling (1929), we assume that a consumer’s product mismatch disutility is a linear function of his location on the Hotelling line.

In particular, when receiving the more suitable product, the disutility is given by $d \cdot \left(\frac{1}{2} - s(\theta)\right) \in [0, d^2]$, and when receiving the less suitable product, the disutility is $d \cdot \left(\frac{1}{2} + s(\theta)\right) \in [d^2, d]$, where $d \in \mathbb{R}^+$ is a scaling parameter.

The consumer seeks to maximize expected utility. If the consumer purchases the policy directly from the insurer, there is a 50 percent chance to select the better suitable contract. Hence, the expected utility of purchasing policy $n \in \{A, B\}$ is given by

$$
EU_{\text{direct}, n}(s(\theta)) = \frac{1}{2} \left[ v - p_n - d \left(\frac{1}{2} - s(\theta)\right) \right] + \frac{1}{2} \left[ v - p_n - d \left(\frac{1}{2} + s(\theta)\right) \right].
$$

If the consumer purchases the policy through a broker, the probability of receiving the better suitable product is $q_i$, as discussed in Section 2.2. Ex ante, it is equally likely that the better suitable contract is either policy $A$ or policy $B$. As a result, when being advised by broker $i$, a consumer’s expected utility is given by

$$
EU_{\text{broker}, i}(s(\theta)) = q_i \cdot \left[ v - (1 + \alpha)p_n + p_m^2 + \gamma_{i,n} + \gamma_{i,m}^2 - d \left(\frac{1}{2} - s(\theta)\right) \right] + (1 - q_i) \cdot \left[ v - (1 + \alpha)(1 - \beta_i)p_n + p_m^2 - d^2 - ds(\theta) + 2ds(\theta)q_i, \right]
$$

13 Following d’Aspremont et al. (1979), it is assumed that consumers are risk-averse with respect to the insured risk and risk-neutral regarding the mismatch risk.

14 As shown by d’Aspremont et al. (1979), a convex disutility function is required to obtain maximum differentiation between insurers in equilibrium, whereas Hotelling’s linear approach does not guarantee that the insurance companies are located at 0 and 1 on the line in the Nash equilibrium. As we concentrate on the equilibrium between customers and brokers and posit that insurers are located at 0 and 1, we can rely on the simpler linear approach. Note that using a quadratic disutility function would not change our findings, as the quadratic disutility terms would cancel out in the Lemmata 2 and 4 below.

15 The disutility $d$ in this model corresponds to the transportation costs $t$ in Hotelling (1929).

2.3 Consumers

Consumers are price takers. They draw positive utility, represented by the monetary equivalent $v$, from covering their financial risk via either of the two policies. Yet, both policies are not equally attractive to the individual consumers, due to their particular preferences for contract characteristics. Diverging consumer preferences are modeled using the Hotelling line (see Figure 1). Specifically, consumers are assumed to be uniformly distributed along the preference interval $[\theta, \bar{\theta}] := [0, 1]$, where policy $A$ is located at $\theta$, policy $B$ is located at $\bar{\theta}$, and where a consumer’s $\theta \in [\theta, \bar{\theta}]$ measures how suitable policies $A$ and $B$ are for that individual.[11] That is, for consumers with $\theta$ values close to the endpoints of the interval one policy provides a (much) better fit than the other, while for consumers with $\theta$ around 0.5 both policies are about equally suitable.[12]

10 An observable signal is important because it is fair to assume that broker’s advice quality itself cannot be directly observed by consumers. Hence, without such a signal, consumers would not have a higher willingness to pay for the higher advice quality, and brokers could not charge a higher price to recover their investment. Consequently, in the absence of an observable signal, the market for high-quality advice will fail.

11 Following d’Aspremont et al. (1979), the ‘principle of maximum differentiation’ in spatial competition applies in this context and this assumption is without loss of generality.

12 Alternatively, we could assume that they have specific preferences for $\theta$ values closer to the corners and unspecific preferences for $\theta$ values closer to 0.5.
 Consumers cannot directly observe their $\theta$. Instead, a consumer located at $\theta$ only observes a signal $s(\theta) = |\theta - \frac{1}{2}| \in [0, \frac{1}{2}]$, which describes that individual’s exposure to mismatch risk. Consumers receiving a signal $s(\theta) = 0$ know that they are not exposed to the risk of a potential mismatch. Hence, they are indifferent between the two policies and will purchase a policy directly, as soliciting further (costly) advice from a broker does not increase their utility. By contrast, consumers receiving a signal closer to $s(\theta) = \frac{1}{2}$ know that one of the policies is (much) more suitable than the other, however, they don’t know which policy is the better one. Hence, these consumers are highly exposed to mismatch risk, which can make asking a broker for additional advice attractive.

Following Hotelling (1929), we assume that a consumer’s product mismatch disutility is a linear function of his location on the Hotelling line. In particular, when receiving the more suitable product, the disutility is given by $d \cdot (\frac{1}{2} - s(\theta)) \in [0, \frac{d}{2}]$, and when receiving the less suitable product, the disutility is $d \cdot (\frac{1}{2} + s(\theta)) \in [\frac{d}{2}, d]$, where $d \in \mathbb{R}^+$ is a scaling parameter.

The consumer seeks to maximize expected utility. If the consumer purchases the policy directly from the insurer, there is a 50 percent chance to select the better suitable contract. Hence, the expected utility of purchasing policy $n \in \{A, B\}$ is given by

$$EU_{direct,n}(s(\theta)) = \frac{1}{2} [v - p_n - d \left(\frac{1}{2} - s(\theta)\right)] + \frac{1}{2} [v - p_n - d \left(\frac{1}{2} + s(\theta)\right)].$$ (2.1)

If the consumer purchases the policy through a broker, the probability of receiving the better suitable product is $q_i$, as discussed in Section 2.2. Ex ante, it is equally likely that the better suitable contract is either policy $A$ or policy $B$. As a result, when being advised by broker $i$, a consumer’s expected utility is given by

$$EU_{broker,i}(s(\theta)) = q_i \cdot \left[v - (1 + \alpha) \frac{p_n + p_m}{2} + \frac{\gamma_{i,n} + \gamma_{i,m}}{2} - d \left(\frac{1}{2} - s(\theta)\right)\right] + (1 - q_i) \cdot \left[v - (1 + \alpha) \frac{p_n + p_m}{2} + \frac{\gamma_{i,n} + \gamma_{i,m}}{2} - d \left(\frac{1}{2} + s(\theta)\right)\right]$$

$$= v - (1 + \alpha(1 - \beta_i)) \frac{p_n + p_m}{2} - \frac{d}{2} - ds(\theta) + 2ds(\theta)q_i,$$ (2.2)

---

13 Following d’Aspremont et al. (1979), it is assumed that consumers are risk-averse with respect to the insured risk and risk-neutral regarding the mismatch risk.

14 As shown by d’Aspremont et al. (1979), a convex disutility function is required to obtain maximum differentiation between insurers in equilibrium, whereas Hotelling’s linear approach does not guarantee that the insurance companies are located at 0 and 1 on the line in the Nash equilibrium. As we concentrate on the equilibrium between customers and brokers and posit that insurers are located at 0 and 1, we can rely on the simpler linear approach. Note that using a quadratic disutility function would not change our findings, as the quadratic disutility terms would cancel out in the Lemmata and below.

15 The disutility $d$ in this model corresponds to the transportation costs $t$ in Hotelling (1929).
where \( n, m \in \{A, B\} \) and \( n \neq m \).

With that, the consumer’s objective function is

\[
\max(E_{\text{direct}, A}, E_{\text{direct}, B}, E_{\text{broker}, 1}, E_{\text{broker}, 2}).
\] (2.3)

2.4 Game Structure

The timeline of the game between insurers, brokers and consumers is as follows: First, the commission rate factor \( \alpha \in (0, 1) \) is exogenously given. Then, firms decide on prices \( p_A \) and \( p_B \) and brokers decide whether or not they improve their advice quality (from \( q_L \) to \( q_H \)). Brokers can then decide on kickbacks \( \gamma_{i,n} \) to consumers. A mismatch risk \( s(\theta) \) is signaled to consumers, and consumers decide whether to take advice (and which one) or to choose to purchase a product directly from an insurer. Next, consumers’ type is signaled to brokers according to advice quality \( q_i \). Brokers give advice according to the signaled type and consumers purchase the advised product.

3 Market Analysis

3.1 Quality Competition without Kickbacks

In this section, it is assumed that kickbacks to consumers are prohibited so that there is a pure quality competition between the brokers \((\beta_i = 0 \text{ and respectively } \gamma_{i,n} = 0)\). Consumers can purchase a product directly or consult one of the brokers. Their decision depends on their individual cutoff-mismatch risk (and therefore on the offered quality) and the prices (see Figure 2).

Consumers who purchase a policy directly from an insurance company are uncertain which of the policies suits their specific needs better, and therefore they will purchase the less expensive one. Bertrand competition then yields zero profits in equilibrium.\(^{16}\)

**Lemma 1.** In the case of direct distribution (without intermediation), the insurance premium equals marginal cost: \( p_A = p_B = p_D = c \).

**Lemma 2.** Consumers with a mismatch risk

\[
s(\theta) \leq s_D = \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_i - 1)}
\] (3.1)

\(^{16}\) All proofs are provided in the appendix.
do not consult a broker but purchase the less expensive product directly.

Note that the higher the advice quality the smaller the cutoff mismatch \( s_D \). It is more worthwhile for the consumer to take advice when the broker offers the higher advice quality.

If one broker is offering \( q_H \) and the other broker \( q_L \), all consumers will consult the high quality broker or purchase a product directly, given that brokers cannot engage in price competition since kickbacks to consumers are prohibited by regulation in the context of this model framework. If both brokers choose the same advice quality, demand is split equally between them. Therefore, brokers’ profits if only broker 1 offers \( q_H \) would be given by

\[
\pi_1(q_H, q_L) = \left( \frac{1}{2} - s_D \right) \alpha p_A + \left( 1 - \left( \frac{1}{2} + s_D \right) \right) \alpha p_B - C_1
\]

\[
= \alpha (p_A + p_B) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_H - 1)} \right) - C_1
\]

and broker 2 would generate no demand (and consequently \( \pi_2(q_H, q_L) = 0 \)). The same advice quality for both brokers implies

\[
\pi_1(q_H, q_H) = \left( \frac{\alpha(p_A + p_B)}{2} \right) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_H - 1)} \right) - C_i
\]

\[
\pi_1(q_L, q_L) = \left( \frac{\alpha(p_A + p_B)}{2} \right) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_L - 1)} \right).
\]

The intuitive question coming to mind is: Under which circumstances is it profitable for a broker to participate in the training? As long as price differentiation is not possible, a low quality broker can never make positive profits when the competitor chooses the high advice quality. Therefore, there are two possible pooling equilibria with either high or low advice quality, depending on the cost structure. However, some consumers won’t need advice and will purchase the product directly from the insurance company (recall Lemma 2). The consumers’ demand for advice is illustrated in Figure 2.

A result about the insurance market equilibrium can now be established. As intuitively expected, the market equilibrium depends on the human capital investment that is necessary in order to obtain the certificate to signal high broker advice quality:

**Proposition 1.** Without kickbacks, the equilibrium advice quality is homogeneous (pooling equilibrium). Three Nash equilibria in pure strategies are possible:

1. If \( C_1 > C^*_1 \),

\[
\frac{\alpha(p_A + p_B)}{4} \left( 1 - \frac{(2q_L - q_A) - (1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_L - 1)(2q_H - 1)} \right),
\]

both brokers offer low advice quality.

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17. That is \( \frac{\partial s_D}{\partial q_1} = -\frac{(1 + \alpha)(p_A + p_B) - 2p_D}{d(2q_1 - 1)} < 0 \).

18. Note that \( dE_U(s(\theta))/dq_1 > 0 \).

19. The reverse holds if only broker 2 increases advice quality.
Figure 2 Schematic structure of advice to consumers.

(2) If \( C_2 < C_2^* \) = \( \left( \frac{\alpha(p_A + p_B)}{2} \right) \left( \frac{1}{2} - \frac{(1+\alpha)(p_A + p_B) - 2p_D}{2d(2q_H - 1)} \right) \), both brokers offer high advice quality.

(3) If \( C_1 \leq C_1^*, \) and \( C_2 \geq C_2^* \), only the efficient broker stays in the market and offers high advice quality.

The market equilibrium is characterized in more detail in the following Lemma:

**Lemma 3.** In the case that kickbacks to consumers are not possible, insurers offer policies at the equilibrium-price \( p^* = \frac{c(3+\alpha) + d(2q_H - 1)}{3(1+\alpha)} \) and gain equilibrium profits of \( \Pi_n(p_n, p_m, s_D) = \frac{(d(1-2q_H) + 2\alpha)^2}{18d(2q_H - 1)(1+\alpha)} \) with \( n \in \{A, B\} \). This holds for sufficiently small marginal cost, \( c < \frac{d(2q_H - 1)}{2\alpha} \). At this equilibrium price brokers earn positive profits.

### 3.2 Quality Competition with Kickbacks

Assume now that kickbacks are permitted, which means that price competition between the brokers can arise in the market. Similar to the above, consumers again will either ask for advice or purchase a product directly. Their decision depends on the individuals’ cutoff mismatch risks.

**Lemma 4.** Consumers with a mismatch risk of \( s(\theta) \leq s_D^* = \frac{(p_A + p_B)[1+\alpha(1-\beta_i)]-2p_D}{2d(2q_H - 1)} \) do not ask for advice and purchase a policy directly from the insurance company.

Since kickbacks to consumers are permitted, quality differentiation can be a useful tool to relax price competition. Especially for the less efficient broker, quality differentiation seems beneficial given that he might otherwise be easily deterred from offering his service by the more efficient broker. In the undifferentiated situation, when both brokers offer high advice quality, the more efficient broker 1 can always undercut broker 2 and force him out of the market, because the maximum kickback broker 1

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20] See, for instance, Gabszewicz and Thisse (1979), or Shaked and Sutton (1982).
21] Recall that the term “less efficient” refers to the higher human capital cost \( C_i \).
high-quality broker 1, as per Lemma 5, the question at hand is whether the low-quality broker 2 is able
to attract any demand. This would require

\[ s^* \cdot D(q_L) \]

and the market to be segmented as illustrated

in Figure 3. The following Lemma concludes that this outcome is not possible:

Lemma 6. If a stable separating \((q_H, q_L)\)-equilibrium existed, brokers would set profit maximizing kick-
backs

\[ \beta^{**}_{1} = 1 + 2(q_H - q_L)[(p_A + p_B) - 2p_D] - d(2q_H - 1)]^{\alpha}\]

\[ \beta^{**}_{2} = 1 + 2(q_H - q_L)[2(p_A + p_B) - d(2q_L - 1)]^{\alpha}\]

This would imply \(\beta^{**}_{1} < 1\) and \(\beta^{**}_{2} > 1\) for all \(q_H, q_L\). Thus, a stable separating equilibrium with positive
broker profits cannot exist, and the low-quality broker 2 will always be forced out of the market.

With only high-quality advice being offered, the mismatch risk above which consumers choose to
purchase a policy from broker 1 is given by (see Lemma 4)

\[ s^*_D(q_H) = (\alpha - \beta_1)(p_A + p_B) - 2p_D - 2d(2q_H - 1)\]

which leads to the following result:

Proposition 2. When kickbacks to consumers are permitted, broker 2 is forced out of the market. The
optimal kickback factor for the high-quality broker 1 is then given by

\[ \beta^*_1 = (p_A + p_B) - d(2q_H - 1) - 2p_D\]

In equilibrium, insurance companies offer policies at

\[ p^*_n = \frac{1}{3}d(2q_H - 1) + c\]

This of course, requires that the investment in human capital is sufficiently low such that one of the
brokers has an incentive to deviate and offer high advice quality. Here, we restrict the analysis to the
case that the efficient broker offers the high quality \((q_1 = q_H)\) and the inefficient broker the low level
of advice quality \((q_2 = q_L)\). In a separating equilibrium, the reversed outcome (where the inefficient
broker offers the high quality and the efficient broker offers the low quality) seems also possible. We do
not further analyze this potential second separating equilibrium since it is economically not sensible.
The rationale is that if the inefficient broker considered offering the high level of advice quality, he
would - at the same time - anticipate being forced out of the market by the more efficient competitor.

Given the different qualities of advice offered in the market, consumers decide which broker to rely on
based on a comparison of the utility they can expect to draw from the consultation, which depends
on the mismatch risk \(s(\theta)\), the disutility \(d\), and the price incentives offered by the brokers through kickbacks \(\beta_1\) and \(\beta_2\). This leads to

Lemma 5. Consumers with mismatch risk

\[ s(\theta) > s^* = \frac{(p_A + p_B)\alpha(\beta_2 - \beta_1)}{4d(q_1 - q_2)} \] (3.2)

consult the high quality broker.

With small mismatch risk resulting in consumers purchasing a product directly from the insurer, as
per Lemma 4, and with high mismatch risk resulting in consumers purchasing a product from the
3.3 Consumer Welfare

In order to analyze how introducing kickbacks affects consumer welfare, we separately assess the implications for three disjunct consumer subgroups: those who purchase directly from the insurer independent of whether kickbacks are offered, those who change the distribution channel when kickbacks are introduced, and those who purchase from the broker in either kickback scenario.

Consumers can always purchase a policy directly from the insurer for a premium $s_D$, independent of whether kickbacks are allowed or not. Hence, for those purchasing directly in either kickback scenario, consumer welfare is not affected by allowing brokers to offer rebates.

Turning to those who switch the distribution channel, we first need to answer the question whether introducing kickbacks increases or decreases the number of consumers interested in purchasing through the broker. For kickbacks to increase the demand for policies sold by the broker, the cutoff mismatch risk $s^*_D$ from Lemma 4 needs to be lower than the cutoff mismatch risk $s_D$ from Lemma 2. As only the high-quality broker 1 is offering policies in the kickback scenario, the condition $s^*_D < s_D$ is given by

$$22\left[\alpha(1 - \beta_1) + 1\right] + 2p_A + 2p_B - 2p_D - 2d(2q_H - 1) < \alpha(1 + \alpha)(p_A + p_B) - 2p_D - 2d(2q_H - 1).$$

which always holds for $\beta_1^* > 0$. Hence, in the presence of kickbacks, more consumers purchase the better-fitting policies through the broker, despite the fact that directly distributed policies are still available at equal conditions. This implies a kickback-related increase in consumer welfare for this subgroup.

Finally, we turn to those who always purchase their policies via the broker. For this subgroup to experience a kickback-induced increase in consumer welfare, net premiums paid in the kickback scenario need to be lower than those paid when kickbacks are not allowed, which requires

$$\left[\frac{\alpha}{3}(1 + \alpha)(p_A + p_B) - 2p_D - 2d(2q_H - 1)\right] < \frac{\alpha}{3}(1 + \alpha)(3 + \alpha) + d(2q_L - 1).$$

Here, we need to differentiate between the high-quality and the low-quality equilibrium in the no-kickback case, as this determines whether $q_L$ on the RHS of (3.8) is equal to $q_H$ or to $q_L$. Substituting $\beta_1^*$ and $p_n^*$ from Proposition 2 and rearranging terms, we can derive the following

$$\beta_1^* = \frac{(p_A + p_B)(1 + 2\alpha) - d(2q_H - 1) - 2p_D}{2\alpha(p_A + p_B)}. \quad (3.6)$$

In equilibrium, insurance companies offer policies at

$$p_n^* = \frac{1}{3}d(2q_H - 1) + c. \quad (3.7)$$
3.3 Consumer Welfare

In order to analyze how introducing kickbacks affects consumer welfare, we separately assess the implications for three disjunct consumer subgroups: those who purchase directly from the insurer independent of whether kickbacks are offered, those who change the distribution channel when kickbacks are introduced, and those who purchase from the broker in either kickback scenario.

Consumers can always purchase a policy directly from the insurer for a premium $s_D$, independent of whether kickbacks are allowed or not. Hence, for those purchasing directly in either kickback scenario, consumer welfare is not affected by allowing brokers to offer rebates.

Turning to those who switch the distribution channel, we first need to answer the question whether introducing kickbacks increases or decreases the number of consumers interested in purchasing through the broker. For kickbacks to increase the demand for policies sold by the broker, the cutoff mismatch risk $s_D^*$ from Lemma 2 needs to be lower than the cutoff mismatch risk $s_D$ from Lemma 2. As only the high-quality broker $1$ is offering policies in the kickback scenario, the condition $s_D^* < s_D$ is given by

$$
\frac{(\alpha(1-\beta_1^*) + 1)(p_A + p_B) - 2p_D}{2d(2q_H - 1)} < \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_H - 1)}
$$

which always holds for $\beta_1^* > 0$. Hence, in the presence of kickbacks, more consumers purchase the better-fitting policies through the broker, despite the fact that directly distributed policies are still available at equal conditions. This implies a kickback-related increase in consumer welfare for this subgroup.

Finally, we turn to those who always purchase their policies via the broker. For this subgroup to experience a kickback-induced increase in consumer welfare, net premiums paid in the kickback scenario need to be lower than those paid when kickbacks are not allowed, which requires

$$
(1 + \alpha(1 - \beta_1^*)) \left( \frac{1}{3} d(2q_H - 1) + c \right) < (1 + \alpha) \frac{c(3 + \alpha) + d(2q_L - 1)}{3(1 + \alpha)} \quad (3.8)
$$

Here, we need to differentiate between the high-quality and the low-quality equilibrium in the no-kickback case, as this determines whether $q_i$ on the RHS of (3.8) is equal to $q_H$ or to $q_L$. Substituting $\beta_1^*$ and $p_n^*$ from Proposition 2 and rearranging terms, we can derive the following

22 For ease of notation, we base the comparison on $s_D$ from the high-quality equilibrium in the no-kickback scenario. As $s_D$ in the low-quality equilibrium is greater than in the high-quality equilibrium, this assumption is not critical for the outcome of the comparison.
Lemma 7. For consumers buying policies through the broker, net premiums in the kickback scenario are lower than in the no-kickback scenario if:

(1) $2\alpha c < d(2q_H - 1) < 4\alpha c$, for the high-quality equilibrium without kickbacks,

(2) $5d(2q_H - 1) - 4d(2q_L - 1) < 4\alpha c < 2d(2q_L - 1)$, for the low-quality equilibrium without kickbacks.

These conditions can be shown numerically to hold for a wide range of reasonable parameter constellations.

4 Conclusion

This paper analyzes the impact of allowing kickbacks (or rebates) on broker commissions in an insurance market where brokers can choose whether to offer high or low advice quality to consumers. Specifically, we employ Hotelling’s (1929) spatial competition approach to analyze market outcomes when consumers face the risk of purchasing a policy less suitable for their individual coverage needs.

Our results indicate that when kickbacks are banned, the market is characterized by a pooling equilibrium, in which either all brokers offer low advice quality or all brokers offer high advice quality. At the same time, independent of which equilibrium results, distribution via brokers always co-exists with direct distribution via insurance companies. By contrast, when kickbacks are allowed, no separating equilibrium exists in which both levels of advice quality are offered simultaneously, and the low-quality broker is always forced out of the market. In this case, only the high-quality broker competes with the direct distributors. Moreover, the market equilibrium with kickbacks results in higher consumer welfare than the equilibrium without kickbacks.

Due to the simplicity of the model, our analysis is subject to several qualifications: First, the brokers are assumed to act non-strategically and not unethically by always recommending the product which, according to their signal, better matches the consumers’ needs. Second, only two levels of advice quality were examined whereas a variety of qualities certainly exists in real insurance markets. Third, the consumers’ demand curves and risk preferences were not explicitly modeled. Fourth, as discussed in Hilliard et al. (2013), resale price maintenance restrictions often prevent brokers from selling policies at lower prices than stated by the insurer (with commissions embedded in the retail price) for reasons that include discouraging brokers from unnecessarily replacing policies to increase commissions, requiring agents to provide customers with greater information services, and to prevent undermining customer persistency. Finally, our model cannot speak to any dynamic aspects, such as policyholders lapsing their policies or the impact of renewal negotiations. We leave augmenting our model with elements that help to overcome the limitations discussed above to future research.
References


Appendix

Proof of Lemma 2:

A consumer's decision between asking for advice from a broker or purchasing directly involves the following expected utilities:

$$
EU_{direct}(s(\theta)) = \frac{1}{2} \left[ v - p_D - d\left(\frac{1}{2} - s(\theta)\right)\right] + \frac{1}{2} \left[ v - p_D - d\left(\frac{1}{2} + s(\theta)\right)\right]
$$

$$
EU_i(s(\theta)) = q_i \left[ v - (1 + \alpha)p_A + p_B^2 - d\left(\frac{1}{2} - s(\theta)\right)\right] + (1 - q_i) \left[ v - (1 + \alpha)p_A + p_B^2 - d\left(\frac{1}{2} + s(\theta)\right)\right]
$$

The cutoff mismatch risk $s_D$, where the consumer is indifferent between the two decisions, is given where

$$
EU_{direct}(s_D) = EU_i(s_D)
$$
or, equivalently,

$$
s_D = (1 + \alpha)(p_A + p_B^2) - 2p_D^2d(2q_i - 1)
$$

(4.1)

As a result, consumers with a mismatch risk of $s(\theta) \leq s_D$ do not consult a broker but purchase the insurance policy directly.

Proof of Proposition 1:

For $(q_L, q_L)$ to be an equilibrium, no profit from deviating should be possible. If only the more efficient broker increases advice quality to $q_H$, the low quality broker will not receive any demand.

The $(q_L, q_L)$-equilibrium is stable if and only if

$$
\pi_i(q_k, q_l) < \pi_i(q_L, q_L)
$$

with $i \in \{1, 2\}$, $k, l \in \{H, L\}$, $k \neq l$.

(4.2)

If inequality (4.2) holds for broker 1, it holds for broker 2, as well, due to $C_1 < C_2$.

$$
\pi_1(q_H, q_L) < \pi_1(q_L, q_L)
$$

for $C_i > C^* = \alpha(p_A + p_B^2)$.

In this case, the investment in human capital is not profitable for the brokers and they do not have any incentive to choose $q_H$, and therefore the stable low-quality equilibrium $(q_L, q_L)$ results.

Since $q_i > 1/2$, the denominator is positive and since $p_n \geq p_D$ and $\alpha \in (0, 1)$, the numerator is also positive.

Note that in this situation only one advice quality ($q_H$) is offered in the market, since the inefficient broker is driven out of the market.


Appendix

Proof of Lemma \[2\]

A consumer’s decision between asking for advice from a broker or purchasing directly involves the following expected utilities:

\[
EU_{direct}(s(\theta)) = \frac{1}{2} \left[ v - p_D - d \left( \frac{1}{2} - s(\theta) \right) \right] + \frac{1}{2} \left[ v - p_D - d \left( \frac{1}{2} + s(\theta) \right) \right]
\]

\[
EU_i(s(\theta)) = q_i \left[ v - (1 + \alpha) \frac{p_A + p_B}{2} - d \left( \frac{1}{2} - s(\theta) \right) \right]
+ (1 - q_i) \left[ v - (1 + \alpha) \frac{p_A + p_B}{2} - d \left( \frac{1}{2} + s(\theta) \right) \right]
\]

The cutoff mismatch risk \( s_D \), where the consumer is indifferent between the two decisions, is given where \( EU_{direct}(s_D) = EU_i(s_D) \) or, equivalently,

\[
s_D = \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_i - 1)}. \tag{4.1}
\]

As a result, consumers with a mismatch risk of \( s(\theta) \leq s_D \) do not consult a broker but purchase the insurance policy directly. \[2\]

Proof of Proposition \[7\]

For \((q_L, q_L)\) to be an equilibrium, no profit from deviating should be possible. If only the more efficient broker increases advice quality to \(q_H\), the low quality broker will not receive any demand. \[24\] The \((q_L, q_L)\)-equilibrium is stable if and only if

\[
\pi_i(q_k, q_l) < \pi_i(q_L, q_L) \quad \text{with} \quad i \in \{1, 2\}, \quad k, l \in \{H, L\}, \quad k \neq l. \tag{4.2}
\]

If inequality (4.2) holds for broker 1, it holds for broker 2, as well, due to \(C_1 < C_2\). \(\pi_1(q_H, q_L) < \pi_1(q_L, q_L)\) for

\[
C_i > C_i^* = \frac{\alpha(p_A + p_B)}{4} \left( 1 - \frac{(2[2q_L - q_H] - 1) \cdot ((1 + \alpha)(p_A + p_B) - 2p_D)}{d(2q_L - 1)(2q_H - 1)} \right).
\]

In this case, the investment in human capital is not profitable for the brokers and they do not have any incentive to choose \(q_H\), and therefore the stable low-quality equilibrium \((q_L, q_L)\) results.

\[23\] Since \(q_i > 1/2\), the denominator is positive and since \(p_A \geq p_D\) and \(\alpha \in (0, 1)\), the numerator is also positive.

\[24\] Note that in this situation only one advice quality \((q_H)\) is offered in the market, since the inefficient broker is driven out of the market.
However, if inequality \( [4.2] \) does not hold for broker 1, deviation to \( q_H \) may be worthwhile; it then depends on the degree of broker 2’s inefficiency whether broker 1 can deter broker 2 or whether the \((q_H, q_H)\)-equilibrium can be obtained.

If \( \pi_2(q_H, q_H) > 0 \), the \((q_H, q_H)\)-equilibrium results, since maintaining \( q_L \) would imply a market exit for broker 2 and thus a monopolistic situation. The costs \( C_i \) have to be sufficiently small to guarantee positive profits in an high-quality equilibrium. Since \( C_1 < C_2 \), profits are positive for broker 1 when they are positive for broker 2. Consequently, for \( C_2 < C_2^* = \left( \frac{\alpha(p_A + p_B)}{2} \right) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_A + p_B) - 2p_D}{2d(2q_H - 1)} \right) \) the high-quality equilibrium arises.

Note that \( C_1^* > C_2^* \) holds due to \( C_2^* - C_1^* = -\frac{\alpha(p_A + p_B)(q_H - q_L)(1 + \alpha(p_A + p_B) - 2p_D)}{2d(2q_H - 1)(2q_L - 1)} < 0 \). For sufficiently low cost \( C_2 \leq C_2^* \), the \((q_H, q_H)\)-equilibrium is stable. For sufficiently large costs \( C_1 > C_1^* \), the \((q_L, q_L)\)-equilibrium is stable. If \( C_1 \leq C_1^* \), and \( C_2 \geq C_2^* \), only the efficient broker stays in the market and offers high advice quality.

**Proof of Lemma [3]**

Insurance firm \( n \), \( n \in \{A, B\} \), sells its policy either directly or through a broker. Demand consists of direct sales and sales via brokers. Sales via brokers are given by sales for broker 1 and broker 2. Looking at one broker’s sales (which constitutes half of the demand) gives

\[
D_{n, \text{broker}}(p_n, p_m, s_D) = \left( \frac{1}{2} - s_D \right) q_i + \left( \frac{1}{2} - s_D \right) (1 - q_i) = \frac{1}{2} - s_D.
\]

Direct sales are given by \( D_{n, \text{Direkt}}(p_n, p_m, s_D) = s_D \). Total demand is \( D_{\text{total}} = \frac{1}{2} - s_D + \frac{1}{2} - s_D + s_D + s_D = 1 \). The profit of firm \( n \), \( n \neq m \) is then given by (remember that both firms have a homogeneous cost structure):

\[
\Pi_n(p_n, p_m, s_D) = (p_n - c) \cdot D_{n, \text{broker}}(p_n, p_m, s_D) + (c - c) \cdot D_{n, \text{direct}}(p_n, p_m, s_D)
\]

\[
= (p_n - c) \left( \frac{1}{2} - s_D \right)
\]

\[
= (p_n - c) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_n + p_m) - 2p_D}{2d(2q_i - 1)} \right)
\]

\[
= (p_n - c) \left( \frac{1}{2} - \frac{(1 + \alpha)(p_n + p_m) - 2c}{2d(2q_i - 1)} \right).
\]

Profit maximization for firm \( n \) implies

\[
\frac{\partial \Pi_n}{\partial p_n} = \frac{d(2q_i - 1) - (p_m + 2p_n)(1 + \alpha) + c(3 + \alpha)}{2d(2q_i - 1)} = 0
\]

19
Proof of Lemma 4:
Comparison of the expected utility of direct sales with the expected utility of advice with kickbacks implies setting
\[ EU_{direct} = EU_i, \]
12
\[ v - p_n - d \left( \frac{1}{2} - s(\theta) \right) \]
\[ = v - (1 + \alpha (1 - \beta)) p_n + p_m^2 - d^2 - ds(\theta) + 2ds(\theta)q_i, \]
Analogously to Lemma 2, solving for \( s \) results in
\[ s_D^* = \left( p_A + p_B \right) \alpha (\beta_2 - \beta_1) 4 d (q_1 - q_2) \]
\[ (4.5) \]
Furthermores, the derivative of the profit function with respect to the quality is \( \frac{\partial \Pi_n}{\partial q_n} = \frac{[d(2q_i - 1)]^2 - (2\alpha)^2}{9d(2q_i - 1)^2(1 + \alpha)} \), which implies that the disutility factor \( d \) needs to be sufficiently large, i.e., \( d > 2\alpha c/(2q_1 - 1) \), to ensure that quality investments lead to higher profits.

At this equilibrium-price brokers earn profits of
\[ \pi_i(q_H, q_H) = \left( \frac{\alpha 2 \left( \frac{c(3 + \alpha) + d(2q_H - 1)}{3(1 + \alpha)} \right)}{2} \right) \left( \frac{1}{2} - \frac{(1 + \alpha) 2 \left( \frac{c(3 + \alpha) + d(2q_H - 1)}{3(1 + \alpha)} \right) - 2c}{d(2q_H - 1)} \right) - C_i \]
\[ \pi_i(q_L, q_L) = \left( \frac{\alpha 2 \left( \frac{c(3 + \alpha) + d(2q_L - 1)}{3(1 + \alpha)} \right)}{2} \right) \left( \frac{1}{2} - \frac{(1 + \alpha) 2 \left( \frac{c(3 + \alpha) + d(2q_L - 1)}{3(1 + \alpha)} \right) - 2c}{d(2q_L - 1)} \right) \]

\[ \frac{\partial^2 \Pi_n}{\partial p_n^2} = -\frac{(1 + \alpha)}{d(2q_i - 1)} < 0 \] holds.\[ \Box \]
Proof of Lemma 4:

Comparison of the expected utility of direct sales with the expected utility of advice with kickbacks implies setting $EU_{direct} = EU_i$,

$$\frac{1}{2} \left[ v - p_n - d \left( \frac{1}{2} - s(\theta) \right) \right] + \frac{1}{2} \left[ v - p_n - d \left( \frac{1}{2} + s(\theta) \right) \right] = v - (1 + \alpha(1 - \beta_i)) \frac{p_n + p_m}{2} - \frac{d}{2} - ds(\theta) + 2ds(\theta)q_i,$$

Analogously to Lemma 2, solving for $s$ results in

$$s^*_D(q_i) = \frac{(p_A + p_B)(1 + \alpha(1 - \beta_i)) - 2p_D}{2d(2q_i - 1)} \geq 0. \quad (4.4)$$

Furthermore, it holds that $\frac{\partial s^*_D(q_i)}{\partial q_i} = \frac{2p_D + (p_A + p_B)(\alpha(\beta_i - 1) - 1)}{d(1 - 2q_i)^2} < 0$ and $\frac{\partial s^*_D(q_i)}{\partial \beta_i} = -\frac{\alpha(p_A + p_B)}{2d(2q_i - 1)} < 0$, meaning consumers are more likely to ask for advice when advice quality and/or kickbacks are higher (for $\beta_i \in [0, 1)$). Even when brokers pass on their whole commissions to consumers ($\beta_i = 1$), $s^*_D(q_i) \geq 0$ still holds. This means that some consumers prefer to purchase a product directly even when the advice is free of charge. This is the case if $p_A, p_B > p_D = c$ (note that for $p_A, p_B = p_D$, we have $s^*_D(q_i) = 0$).

Proof of Lemma 5:

Comparison of the expected utility of the high and low quality broker implies setting $EU_1(s(\theta)) = EU_2(s(\theta))$, which results in

$$v - (1 + \alpha(1 - \beta_1)) \frac{(p_A + p_B)}{2} - \frac{d}{2} - ds(\theta) + 2ds(\theta)q_1 = v - (1 + \alpha(1 - \beta_2)) \frac{(p_A + p_B)}{2} - \frac{d}{2} - ds(\theta) + 2ds(\theta)q_2.$$

Now, solving for $s(\theta)$ yields the required cutoff-mismatch risk:

$$s^* = \frac{(p_A + p_B)\alpha(\beta_2 - \beta_1)}{4d(q_1 - q_2)} \quad (4.5)$$

Proof of Lemma 6:

Note that $(q_H, q_H)$ is no equilibrium since the more efficient broker 1 can undercut broker 2 and force him out of the market. Therefore, broker 2 has an incentive to stay at $q_L$ in order to receive any
demand. If both brokers stay with \( q_L \), Bertrand competition would lead to zero profits for both brokers \( \pi_i(q_L, q_L) = 0, i = 1, 2 \). Thus, only broker 1 has an incentive to increase his advice quality to \( q_H \) if costs \( C_1 \) are sufficiently small to guarantee positive profits, i.e., \( \pi_1(q_H, q_L) > \pi_1(q_L, q_L) = 0 \).

In a differentiated situation, brokers make profits of

\[
\begin{align*}
\pi_1(q_H, q_L) &= \left( \frac{1}{2} - s^* \right) \alpha p_A(1 - \beta_1) + \left( 1 - \left( \frac{1}{2} + s^* \right) \right) \alpha p_B(1 - \beta_1) - C_1 \\
&= \alpha(1 - \beta_1)(p_A + p_B) \left( \frac{1}{2} - \frac{(p_A + p_B)\alpha(\beta_2 - \beta_1)}{4d(q_H - q_L)} \right) - C_1 \\
\pi_2(q_H, q_L) &= \alpha(1 - \beta_2)(p_A + p_B)(s^* - s^*_D) \\
&= \alpha(1 - \beta_2)(p_A + p_B) \left( \frac{(p_A + p_B)\alpha(\beta_2 - \beta_1)}{4d(q_H - q_L)} - \frac{(p_A + p_B)(1 + \alpha(1 - \beta_2)) - 2p_D}{2d(2q_L - 1)} \right)
\end{align*}
\]

If \( C_1 < C_1^* = \alpha(1 - \beta_1)(p_A + p_B) \left( \frac{1}{2} - \frac{(p_A + p_B)\alpha(\beta_2 - \beta_1)}{4d(q_H - q_L)} \right) \), broker 1’s profit is positive, and improving his advice quality is worthwhile. Brokers choose kickback factors to maximize their profit. Then the first-order condition for broker 1 reads

\[
\frac{\partial \pi_1}{\partial \beta_1} = \left( p_A + p_B \right) \alpha \cdot \left( (p_A + p_B)\alpha(1 - 2 \beta_1 + \beta_2) - 2d(q_H - q_L) \right) = 0
\]

and solving for \( \beta_1 \) yields

\[
\beta_1 = \frac{(p_A + p_B)\alpha(1 + \beta_2) - 2d(q_H - q_L)}{2(p_A + p_B)\alpha}.
\] (4.6)

The necessary condition for a maximum holds, \( \frac{\partial^2 \pi_1}{\partial \beta_1^2} = \frac{-(p_A + p_B)^2\alpha^2}{2d(q_H - q_L)} < 0 \).

Turning to broker 2, the first-order condition is

\[
\frac{\partial \pi_2}{\partial \beta_2} = (p_A + p_B) \alpha \left( \frac{(p_A + p_B) \left( \alpha(2q_L - 1)(1 + \beta_1) + 2(q_H - q_L)(1 + 2\alpha) + 2\alpha(1 - 2q_H)\beta_2 \right) - 4p_D(q_H - q_L)}{4d(q_H - q_L)(2q_L - 1)} \right) \hat{=} 0
\]

and solving for \( \beta_2 \) yields

\[
\beta_2 = \frac{(p_A + p_B)[q_H(2 + 4\alpha) + 2q_L(\alpha(\beta_1 - 1) - 1) - \alpha(1 + \beta_1)] - 4p_D(q_H - q_L)}{2\alpha(p_A + p_B)(2q_H - 1)}.
\] (4.7)

The necessary condition for a maximum holds, \( \frac{\partial^2 \pi_2}{\partial \beta_2^2} = \frac{-(p_A + p_B)^2\alpha^2(2q_H - 1)}{2d(q_H - q_L)(2q_L - 1)} < 0 \).
Inserting then (4.7) into (4.6) yields the optimal kickback factor for broker 1,

\[ \beta_1^{**} = 1 + \frac{2(q_H - q_L)(p_A + p_B - 2p_D) - 2d(2q_H - 1)}{\alpha(p_A + p_B)(2(4q_H - q_L) - 3)} \]  

(4.8)

and inserting this \( \beta_1^{**} \) in (4.7) yields the optimal kickback factor for broker 2:

\[ \beta_2^{**} = 1 + \frac{2(q_H - q_L)(2(p_A + p_B - 2p_D) - d(2q_L - 1))}{\alpha(p_A + p_B)(2(4q_H - q_L) - 3)} \]  

(4.9)

To ensure positive broker profits in equilibrium, we must have \( \beta_1^{**}, \beta_2^{**} \in [0, 1] \). To evaluate equilibrium kickback factors \( \beta_1^{**}, \beta_2^{**} \), we first determine equilibrium prices. Insurance firm \( n, n \in \{A, B\} \), sells insurance both directly and via the brokers. Sales via brokers are given by

\[
D_{n,\text{broker}}(p_n, p_m, s_D^*, s^*) = \left( \frac{1}{2} - s^* \right) q_H + \left( 1 - \left( \frac{1}{2} + s^* \right) \right) (1 - q_H) \\
\text{appropriate advice broker}_1 + \text{inappropriate advice broker}_1 \\
+ \left( s^* - s_D^* \right) q_L + \left( s^* - s_D^* \right) (1 - q_L) \\
\text{appropriate advice broker}_2 + \text{inappropriate advice broker}_2 \\
= \left( \frac{1}{2} - s^* \right) + (s^* - s_D^*).
\]

The profit of firm \( n \) (\( n \neq m \)) is

\[
\Pi_n(p_n, p_m, s_D^*, s^*) = (p_n - c) \cdot D_{n,\text{broker}}(p_n, p_m, s_D^*) + (c - s_D^*) \cdot D_{n,\text{direct}}(p_n, p_m, s_D) \\
= (p_n - c) \left( \frac{1}{2} - s^* \right) + (s^* - s_D^*) \\
= \frac{1}{2} (p_n - c) \left( 1 + \frac{(p_n + p_m)(\alpha(\beta_2 - 1) - 1) + 2p_D}{d(2q_L - 1)} \right)
\]

respectively profit with optimal kickback factor \( \beta_i^{**} \) (\( i = 1, 2 \))

\[
\Pi_n(p_n, p_m, s_D^*, s^*) = \frac{(c - p_n)[3d(2q_H - 1)(2q_L - 1) + (2c - (p_n + p_m))(4q_H + 2q_L - 3)]}{2d(2q_L - 1)(8q_H + 2q_L - 3)}
\]

Profit maximization for firm \( n \) implies

\[
\frac{\partial \Pi_n}{\partial p_n} = \frac{3[3c - p_n - 2p_D + d(4q_H - 2)]}{8q_H - 2q_L - 3} + \frac{3c - p_n - 2p_D}{2q_L - 1} \equiv 0
\]
and solving for \( p_n \) yields\(^{26}\)

\[
p_n = \frac{1}{2} \left( 3c - p_m + \frac{3d(2q_H - 1)(2q_L - 1)}{4q_H + 2q_L - 3} \right).
\] (4.10)

Analogously, profit maximization for firm \( m (m \neq n) \) yields \( p_m \). Inserting \( p_m \) in \( p_n \) (Eq. (4.10)) and solving for \( p_n \) yields the equilibrium price

\[
p_n^* = c + \frac{d(2q_H - 1)(2q_L - 1)}{(4q_H + 2q_L - 3)} > c,
\] (4.11)

and repeated insertion yields \( p_n^* = p_m^* = p^* \). As a result, insurance firms offer their policies at the same price \( p^* \). Hence, the firms will make profits of \( (n \in \{A, B\}) \)

\[
\Pi_n(p_n^*, p_m^*, s_D, s^*) = \frac{d(2q_H - 1)^2(2q_L - 1)}{2(8q_H - 2q_L - 3)(4q_H + 2q_L - 3)} > 0.
\]

At the equilibrium-price \( p^* \), we will show in the following that \( \beta_1^{**} < 1 \) and \( \beta_2^{**} > 1 \) for all \( q_H, q_L \), and thus a stable separating equilibrium with positive broker profits will not occur. First, for \( \beta_1^{**} < 1 \), the numerator in (4.8) needs to be negative. As \( q_H > q_L \) and \( p_A = p_B = p_n^* \), this requires

\[
2p_n^* - 2p_D < 2d(2q_H - 1).
\] (4.12)

Replacing \( p_n^* \) by the expression in (4.11) and noting that \( c = p_D \), as per Lemma\(^1\) we have

\[
2 \cdot \frac{d(2q_H - 1)(2q_L - 1)}{4q_H + 2q_L - 3} < 2d(2q_H - 1),
\]

which reduces to

\[
\frac{2q_L - 1}{4q_H + 2q_L - 3} < 1
\]

and further to

\[
q_H > \frac{1}{2},
\]

which is always true by construction.

\(^{26}\) Note that the necessary condition for a maximum holds \( \frac{\partial^2 \Pi_n}{\partial p_n^2} = \frac{4q_H + 2q_L - 3}{d(2q_H - 1)(2q_L - 3 - 8q_H)} < 0 \).
Second, we show by contradiction that $\beta_2^{**} > 1$. For $\beta_2^{**} < 1$, the numerator in (4.9) needs to be negative. Again noting that $q_H > q_L$ and $p_A = p_B = p_n^*$, we need to show that

$$2(2p_n^* - 2p_D) < d(2q_L - 1).$$

Using (4.11), we get

$$2 \left( \frac{d(2q_H - 1)(2q_L - 1)}{4q_H + 2q_L - 3} \right) < d(2q_L - 1),$$

which reduces to

$$2q_H - q_L < \frac{1}{2},$$

which is never true, since $\frac{1}{2} < q_L < q_H$ by construction. Thus, no stable separating equilibrium exists.

\textbf{Proof of Proposition 2:}

Broker 1’s profits are given by

$$\pi_1(\beta_1) = \left( \frac{1}{2} - s_D^* \right) \alpha p_A (1 - \beta_1) + \left( 1 - \left( \frac{1}{2} + s_D^* \right) \right) \alpha p_B (1 - \beta_1) - C_1,$$

and, inserting $s_D^*$, can be rewritten as

$$\pi_1(\beta_1) = \frac{1}{2} \alpha(1 - \beta_1)(p_A + p_B) - \frac{\alpha(p_A + p_B)^2(1 - \beta_1)(1 + \alpha(1 - \beta_1))}{2d(2q_H - 1)} + 2\frac{\alpha(1 - \beta_1)(p_A + p_B)p_D}{2d(2q_H - 1)} - C_1,$$

which, by taking the derivative and setting equal to zero, results in an optimal $\beta_1^*$ of

$$\beta_1^* = \frac{(p_A + p_B)(1 + 2\alpha) - d(2q_H - 1) - 2p_D}{2\alpha(p_A + p_B)}.$$

The second order condition for a profit maximum is met. Inserting the above into the insurers’ profit function, and taking the derivative with respect to $p_n$ results in

$$\frac{\partial \Pi_n}{\partial p_n} = \frac{1}{2} + \frac{0.5(2p_n - c + p_m)(1 + 2\alpha) - 0.5d(2q_H - 1) - p_d - \alpha(2p_n - c + p_m) - (2p_n - c + p_m) + 2p_D}{2d(2q_H - 1)}.$$
and setting equal to zero leads to

$$p_n^* = 0.5d(2q_H - 1) + p_D + 0.5(c - p_m) \quad (4.16)$$

The second order condition is met. In equilibrium, we must have $p_n = p_m$ as well as $p_D = c$, thus

$$p_n^* = \frac{1}{3}d(2q_H - 1) + c. \quad (4.17)$$

It remains to be verified that $\beta_1^* < 1$ in equilibrium. Using $\beta_1^*$ from (4.17), we need to show that

$$\frac{(p_A + p_B)(1 + 2\alpha) - d(2q_H - 1) - 2p_D}{2\alpha (p_A + p_B)} < 1, \quad (4.18)$$

which can be rearranged to

$$(p_A + p_B)(1 + 2\alpha) - d(2q_H - 1) - 2p_D < 2\alpha(p_A + p_B)$$

and reduced to

$$(p_A + p_B) - d(2q_H - 1) - 2p_D < 0. \quad (4.19)$$

With $p_A = p_B = p_n^*$ and $p_D = c$, substituting (4.19) into (4.25) results in

$$\frac{2}{3}d(2q_H - 1) + 2p_D - d(2q_H - 1) - 2p_D < 0, \quad (4.20)$$

which reduces to

$$-\frac{1}{3}d(2q_H - 1) < 0, \quad (4.21)$$

which always holds.

**Proof of Lemma 7:**

As per (3.8), net premiums paid in the kickback scenario are lower than those paid when kickbacks are not allowed if

$$(1 + \alpha(1 - \beta_1^*)) \left( \frac{1}{3}d(2q_H - 1) + c \right) < (1 + \alpha) \frac{c(3 + \alpha) + d(2q_H - 1)}{3(1 + \alpha)}, \quad (4.22)$$