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The Sub-prime Mortgage Debacle and What We Can Learn From Mathematical Programs

Strategic Analysis: Approaching Continuous Improvement Proactively

Assessing Redundancy’s Impact on the Reliability of Microcontroller/Processor-Based Systems in Mission Critical Applications

Problems with the Application of a Metric for CRA Compliance in Banking

The Intuition and Methodology of Value at Risk

Input-Output Methodology used for Forecasting Purposes — A Cost Analysis

The Role of Modeling in Scientific Disciplines: A Taxonomy

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From the Editor

The Review of Business has passed another milestone. 2008 is the Journal’s 40th year of operation! It started in 1968 under the editorial helm of Dr. Francis Lees of the Economics and Finance Department, as a newsletter focused on business activities of our Queens County. Thank you Dr. Lees, and many thanks to a series of good editors that brought the Journal to the level it has attained, which enabled it to win the AUBER Award in 1998 for quality in publication. I am very grateful to Larry Boone, Brenda Massetti, and our Associate Editor Maxine Brady for their assistance as I took on the new task as editor of this Journal.

The main focus of the Journal on applied research will remain the same, although other avenues of research will not be excluded. In time, the scope of articles will become more global in their coverage; a reflection of the needs of our readership and the increasing impact of how the influence of policies in one country affects others ever more directly.

This is a special issue that was produced with the assistance of a guest editor, Dr. F. Victor Lu. Dr. Lu is the chair of the CIS/DS Department of the Tobin College of Business, and he will shed some more light on the content of this issue.

Dr. Igor M. Tomic
Editor, Review of Business

From the Special Issue Editor

This is a special issue organized to showcase a range of interesting and successful applications in the field of Decision Sciences for the Review of Business. The Decision Sciences discipline applies a scientific approach, and mathematical modeling methodology on computers, to yield solutions in supporting a wide spectrum of decision-making activities. In the eyes of professionals, it is a field that is connected almost interchangeably with the fields of Management Science, Operations research, Information Science and Quantitative Analysis. The growing popularity of the Decision Sciences discipline may be evidenced by an article, “Higher Math Delivers Formula for Success — Businesses Turn to Algorithms to Solve Complex Problems,” that was headlined in the Money section of USA Today on December 31, 1977. According to U.S. Labor Department projections, the demand for Decision Sciences and Information Systems professionals is expected to grow dramatically in the next 10 years.

Given the recent and ongoing sub-prime mortgage debacle, it is imperative that the conscientious and conservative investor use robust mathematical models instead of investing in financial instruments using a “seat of the pants” approach. In their timely article “The Sub-prime Mortgage Debacle and What We Can Learn from Mathematical Programs,” Robert Fireworker, Gavriel Yarmish and Harry Nagel discuss an approach to value a mortgage-backed security, and methods for using these programs for asset allocation. The authors first describe the difference between a deterministic and stochastic instrument, and then describe a linear programming model for allocating bonds deterministically and then, as in the case of mortgage-backed securities,
From the Editor

stochastically. These models are suitable for the conservative investors who would like to take advantage of securities that offer decent returns, but who at the same time need to be sure that their obligations are met. The paper also provides a number of real-world business examples, illustrating the applicability of stochastic linear programming techniques to both past and current real-world practices.

For an organization to survive in today’s complex international business environment, it must constantly evaluate how well it is functioning, and make whatever changes are necessary in response to new challenges. These tasks must be carried out with no disruption in corporate productivity. In “Strategic Analysis: Approaching Continuous Improvement Proactively,” Lisa B. Ncube and Mara H. Wasburn discuss the need for an ongoing assessment process to identify and clarify the challenges facing an organization. Using Strategic Analysis, the company can then determine what courses of action are available to it, to confront these challenges most effectively. This is an ongoing process for continuous corporate improvement.

Reliability is defined as the ability of a system or component to perform its required functions under stated conditions for a specified period of time, and it is often reported or expressed in terms of a probability. In his paper “Assessing Redundancy’s Impact on the Reliability of Microcontroller/Processor-Based Systems in Mission Critical Applications,” Michael I. Liechenstein analyzes the absolute and relative mission reliability of the microcontroller/processor-based systems as a function of redundancy level for linked, parallel units operating in either inactive or active standby modes under constant hazard rates. The paper studies two dynamical models — inactive standby units and active redundant units — and it reveals noteworthy insights regarding the impact of redundancy on overall system mission reliability.

In 1977, the U. S. Congress passed the Community Reinvestment Act (CRA) with an aim intended to reduce discriminatory credit practices against low- and moderate-income neighborhoods, a practice known as ‘redlining.’ CRA is politically motivated and is designed to encourage banking institutions to meet the needs of borrowers in all segments of their communities. Specifically, federal regulatory bodies can use the degree of CRA compliance to approve or disapprove the applications for new branches, or for mergers or acquisitions of banks. In his paper “Problems with the Application of a Metric for CRA Compliance in Banking,” Andrew Russakoff discusses some of the ways that the government has attempted to implement CRA compliance, what this reveals about the implicit model for investment, and how the metric for CRA compliance might be modified to better address the issue. He reveals that the creation of a metric for CRA compliance is not an easy task due to intriguing political factors or influences. In particular, he uses the two models (Treatment Effects and Probit Model) suggested by Dahl, Evanoff, and Spivey (for the Federal Reserve Bank) for detecting the sensitivity of bank mortgage loans to CRA down grading. This is an important example because it may reveal the weakness of introducing measurement standards in a situation which straddles two very different sets of values: the economic and the political.

Faced with the unprecedented debacles of large financial institutions, the severe freezing of credit markets and huge losses of investors’ pensions and investment portfolios, Manuel Russon’s paper on measuring the Value at Risk (VaR) of an investment portfolio is timely and valuable. Value at Risk (VaR) was introduced in the 1980’s and is defined as the largest portfolio loss that could be sustained in any given period of time for a given level of confidence. The purpose of VaR is to provide an analyst, an executive, a risk manager, or a regulator with a guide to appraise
the risk of an enterprise at any particular moment, as well as the trends in the risk of an entire enterprise. In his paper, “The Intuition and Methodology of Value at Risk,” Russon presents four different methods used to compute the magnitude of VaR for a given of confidence, each with varying degrees of complexity. The four methods are: Historical VaR and Historical Parametric, Parametric VaR, Simulated Historical VaR and Simulated Parametric VaR. Each method relies on a set of historical data as inputs for the computations and the difference between the methods lies in what is done with the historical data to arrive at a VaR. It is evident that VaR is an attempt by management to model human behavior as instigated by real economic, geopolitical, environmental phenomena affecting markets and it amounts to sophisticated guesswork. The author cautions about the use of applying VaR to measure a portfolio or an enterprise as it has limitations due to its heuristic nature, and is subject to attendant assumptions and complications as represented by skewness, leptokurtosis and outliers (especially), and conditional volatility and conditional correlation.

Input-output analysis investigates inter- or cross-industry relations in an economy by studying how the output of one industry is used by another industry where it serves as an input. In essence, industries interact with each other, with each industry acting both as customer of output and as supplier of inputs. In his paper “Input-Output Methodology Used for Forecasting Purposes - A Cost Analysis,” A. Vasilopoulos proposes a methodology for extending the static (open-loop) model to a dynamic and recursive model that are suitable for forecasting. A cost function is defined and derived for the basic and extended models and shown to be “relatively reasonable” when compared to the cost of the static model, thus making the Input-Output methodology a useful forecasting tool.

Friedman, Friedman and Pollack investigate what scientists in each discipline can learn from models of other fields. They create a framework that will go a long way towards providing a structure for further interdisciplinary scholarly work; a way for scholars from different areas of study to establish common language and thereby develop new paradigms for research in this age of disciplinary convergence. “The Role of Modeling in Scientific Disciplines: A Taxonomy” provides an examination of the various models, and reveals that there is indeed some commonality in the way very different fields process information. Each model is a view of reality; it has a purpose; and it employs abstraction, structure and information hiding. In addition, each model alters reality to some degree. Scholars working in one discipline, by understanding how models are used in other areas of study, will be able to develop new types of models and thus ultimately gain a better understanding of their own discipline.

Dr. F. Victor Lu
Special Editor
Chair of CIS/DS Department
The Sub-prime Mortgage Debacle and What We Can Learn from Mathematical Programs

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Abstract

In this paper we discuss mathematical programming methods for insurance companies, mutual fund managers, and banks to match cash flow and liabilities. We focus on mortgage-backed securities, and methods for using them for asset allocation. Given the recent and ongoing sub-prime mortgage debacle, it is imperative that the conscientious and conservative investor use robust mathematical models to decide whether to hold or sell their current investments, or to invest in other portfolios, instead of making such decisions without careful consideration. We first discuss the difference between a deterministic and stochastic instrument, then describe a linear programming model for allocating bonds deterministically, and then, as in the case of mortgage-backed securities, stochastically.

Introduction

The objective of this paper is to learn from our current mortgage crisis to avoid investment without careful consideration and without proper valuation. In particular, this paper is directed at large investors such as insurance companies and mutual funds whose investments are supposed to be conservative and with low risk.

Methodology

We begin by describing the mortgage debacle, how it developed and who was affected. We then focus on the financial objectives of mutual funds and insurance companies. Insurance companies collect large sums of money as premiums. Those premiums are held in financial funds, which require asset allocation decisions. These funds are often governed by laws that aim at keeping the investments conservative and low risk. After describing the general asset allocation problem we explain the difference between a deterministic and stochastic cash flow.

• Section 3: A Deterministic Linear Program: Cash Flow/Liability Matching with Deterministic Bonds, describes a mathematical (linear) program that assumes investment in a bond that has a deterministic (known) cash flow. The linear program matches cash inflow with liabilities.
• Section 5: A Stochastic Linear Program: Cash Flow/Liability Matching with Mortgage Backed Securities (MBS) describes a linear program that allows investment in a Mortgage-Backed-Security (MBS), a security that has stochastic (unknown) cash flow. Stochastic instruments do have problems with their past performance, and a probability distribution associated with them.
• The purpose of Section 4: Pricing a Stochastic Instrument, is to introduce the reader to the way a stochastic instrument is valued. This is accomplished by generating many scenarios, Monte Carlo style, based on the probability distribution of each one, and by then taking the Expected Value with respect to all of those scenarios.
• Section 4 can thus be viewed as an introduction to section 5.
• The mathematical programs of sections 3 and 5 are specifically suited to insurance companies and banks that invest their capital but yet have specific obligations that must be met.
• Section 6: More Examples of Stochastic Linear Programs, discusses some examples where mathematical programming has been utilized to match cash inflow and liabilities of complicated real-world situations.

A deterministic instrument is an instrument in which the cash flows are known with certainty. An example of this is a conventional bond.

The Mortgage Debacle

We are in the midst of a mortgage debacle spurred on by rising property values and reckless lending practices. As property values rose, lenders, desirous of capitalizing on the buying frenzy, offered loans to borrowers even if their credit-worthiness was sub-prime.

In order to entice possible borrowers, variable-rate loans with low initial interest rates, also known as “teaser rates,” were offered. These loans offered the borrower low interest payments for the first few years, which would then jump after a number of years. It was recognized that the only way these borrowers would be able to continue with the payments would be through constantly rising property values. Indeed, these easy loans with teaser rates helped drive property values up even higher. The generous lending policies brought many more people into the home-buying market.

In order for banks to offset their risk and to raise immediate cash, lending banks would package these loans into mortgage-backed securities and sell them. In this manner the inherent risk was passed on to investors.

As long as property values rose, the market was able to sustain repayment of these loans. The home-owner was able to either refinance based on higher home values, or he or she would find a buyer at a higher purchase price and thus pay off the loan.

But once the upward swing of property values stopped, the defaults began. A scheme such as this ultimately tumbles in the same manner as any pyramid scheme.

There are a number of losers in this game:
1. Home buyers who lose their homes after buying homes that are beyond their means
2. Investors who bought up the mortgage-backed securities for homes, some of which are moving towards default and some of which have already defaulted and
3. Banks that are left holding many mortgage-backed securities with no buyers.

It is important to note that it was not only risk-taking investors who bought mortgage-backed securities, but also conservative investors; mutual funds, retirement funds, and others. These securities were often given high credit ratings by the agencies upon whom those investors relied. Investors trusted banks to make loans only to credit-worthy borrowers, and they assumed that this package of collateralized securities must be a very safe instrument.

With this debacle in mind it is imperative that mutual funds, retirement funds, insurance
companies and other conservative investors use robust methods, such as mathematical models, to help in asset allocation decisions.

1. The Asset Allocation Problem

The financial Asset Allocation Problem considers the question of how a portfolio should be weighted with different security assets in order to satisfy an investor’s objective (Markowitz, 1959). Investors’ objectives may differ. For instance, an individual who is saving money for retirement will want to be quite sure that the money will be there when it is needed; thus the risk must be minimized.

An expansion of this idea occurs when we build liability and other constraints into the model. For example, an insurance company that sells insurance on a long term basis will want to invest the initial money culled from sales to earn a substantial enough return to enable it to charge competitive prices, while still maintaining confidence in its ability to meet its obligations. A company having obligations due over many years may want to invest, but may not wish to risk the amount necessary for its future obligations.

2. Deterministic vs. Stochastic Instruments

A deterministic instrument is an instrument in which the cash flows are known with certainty. An example of this is a conventional bond. The investor knows that over the course of some time, a set amount, known as the coupon, will be paid for every period specified. This assumes that the cash flow does not come from the sale of the bond. The value of the bond itself does depend on interest rates, and the cash from its sale would not be deterministic.

A stochastic obligation is an obligation in which the cash flows are not known with certainty. An example of this is any stock or a bond that pays a floating rate instead of a fixed rate. That is, the payment is pegged to an outside rate that changes in a non-deterministic way (such as LIBOR). Clearly, if interest rates go down the bond will yield less.

A stochastic obligation is an obligation in which the cash flows are not known with certainty. An example of this is any stock or a bond that pays a floating rate instead of a fixed rate.

Another example of a stochastic obligation is a Mortgage Backed Security (MBS). When a bank wishes to raise capital to lend to homeowners, it may create a security called an MBS, which can be traded. A number of mortgages are associated with one MBS. An investor buys the MBS and the bank shifts the mortgage payments to the investor when they are paid. These payments are stochastic due to the homeowner’s refinancing option. Suppose, for example, that interest rates were to go down so that the homeowner can now borrow at a lower rate of interest. The homeowner may prepay the principal, and as a result, the investor’s cash flow is not deterministic. There are, of course other sources of risk, most notably in the current crisis: risk of default.

3. A Deterministic Linear Program: Cash Flow/Liability Matching with Deterministic Bonds

As just mentioned, a deterministic portfolio of bonds does not depend on changes in interest rates. This may be seen from the following scenario (Schrage, 1994). A pension fund has a list of people to whom pensions must be paid. The fund would collate all the numbers so that they have one aggregate liability due each year for a set number of years. To fund these liabilities, the fund may choose from a specific group of bonds. The question is how to best balance a portfolio of bonds from among their set of bonds.
Let \( n \) = the number of types of bonds in their possible set.
Let \( p \) = the number of periods (in our case, years) to which the liabilities extend.
Let \( C_{ij} \) = the coupon rate bond \( i \) pays in year \( j \).
For some years \( j \), bond \( i \) may be past maturity and \( C_{ij} = 0 \). \( C_{ij} \) for all \( j \)'s less than bond \( i \)'s maturity year are equal.
Let \( L_j \) = the liability due in year \( j \).
Let \( B_i \) = the number of bonds of type \( i \) that we will purchase for our portfolio in time 0.
These are the variables for which we need to solve.
Let \( R \) = the lower bound of the short term interest rate.
Let \( S_j \) = the amount of extra cash reinvested in year \( j \).
Let \( \text{price}_i \) = the cost of bond \( i \).

Note that \( j = 0 \) is before the first year; at the time we run the mathematical program and make our decision.

The idea is to minimize the cost of the portfolio, given that the liabilities will be funded.

**Mathematical and stochastic programs, linear or otherwise, have been used to solve a myriad of problems in addition to those of cash flow/liability matching.**

We allow the reinvestment of extra money each year. This is represented by \( R \) and \( S \) defined above. \( R \) depends on changing interest rates. Therefore a lower bound is picked for \( R \), under which it is assumed interest rates will not fall. Thus the problem remains deterministic. It was necessary to include reinvestment in this model because otherwise the constraints would conflict. In this model the bonds are purchased at the establishment of the portfolio, after which the portfolio is not changed.

There might therefore be a span of years during which the portfolio, comprised of bond coupons, would pay the same amount. The liabilities, on the other hand, can be different from year to year. A method was needed to smooth out portfolio payments so that a year of high payment can be bumped over to help cover next year's liability.

The following is the deterministic Linear Program:

\[
\text{Min } \sum_{i=1}^{n} \text{price}_i B_i + \sum_{j=1}^{p} C_{ij} B_j - S_j = L_j \quad \forall j \geq 1
\]

or

\[
\sum_{i=1}^{n} \text{price}_i B_i + \sum_{j=1}^{p} C_{ij} B_j + RS_{j-1} - S_j = L_j \quad \forall j \geq 1
\]

4. Pricing a Stochastic Instrument

As noted above, Mortgage Backed Securities give cash flows that depend on changes in interest rates. Therefore in order to solve for the price, or present value, of an MBS we must know how interest rates will move. This, of course, is unknown, so we do the next best thing: we estimate their movement (Zenios, 1991; Cagan, Carriero & Zenios, 1993). There are an infinite number of possible ways that interest rates can move. Our strategy is to take as a basis the forward interest rates implied by the current spot treasury curve. First, all forward interest rates for all periods from now until a specified future time, called the horizon, are calculated. These are usually summarized in a forward interest rate curve. Then, using a model which describes how interest rates move, a possible scenario of interest rates from now until the horizon is simulated, based on the calculated forward interest rate curve. This simulation is performed thousands of times to simulate thousands of possible interest rate scenarios. It is apparent that this can be very computer intensive.

Imagine an MBS connected to 30 year mortgages. In order to have small changes in interest rates, rates are simulated monthly. This gives us 360 periods. For 1000, scenarios there
are 360,000 possible ‘random walks’ of interest rates.

Using these scenarios, we estimate at each point whether or not the homeowner would prepay the loan in order to refinance. This can be done in one of two ways. Pre-computed tables bearing this information can be used, or models can be set up for this purpose. After knowing the interest rates of each scenario and whether or not the homeowner will prepay at each point in each scenario, it is relatively straightforward to calculate the cash flows resulting from the MBS for each scenario. For each period, the net interest paid (ni), the scheduled principal paid (p), and the amount of principal prepaid (pre), are known. The cash flow for the period is \( cf = ni + p + pre \). The present value for each scenario over all their cash flows is calculated.

\[
\text{Let } cfm = \text{ the cash flow for month } m.
\]
\[
\text{Let } p = \text{ the number of months in one scenario.}
\]
\[
\text{Let } r_j = \text{the Forward one month Treasury Bill rate for month } j.
\]
\[
\text{Let } oas = \text{the extra return an MBS should get over a treasury bill.}
\]

The MBS present value for one scenario is
\[
\sum_{m=1}^{p} \frac{cf_m}{(1 + r_1 + oas)(1 + r_2 + oas) \ldots (1 + r_m + oas)}
\]
or
\[
\sum_{m=1}^{p} \frac{cf_m}{1 + r_m + oas}
\]

Note that the cash flows are discounted by the monthly treasury bill rate plus an additional percentage. This is because there is some risk due to the possibility of the homeowner’s refinancing. This means that an MBS should provide a higher return than treasury bills and should similarly be discounted by that rate.

Then we took the expected value of the price over all scenarios by calculating the value for each scenario as above, weighting it with the scenario’s probability, and then summing them together.

5. A Stochastic Linear Program: Cash Flow/Liability Matching with Mortgage Backed Securities (MBS)

Thus far we have shown how a bond that gives uncertain cash flows can be valued. This method used the simulation of a random variable, namely interest rates. This was required due to the stochastic nature of an MBS. Our simulation took scenarios of probable interest rate streams.

An expansion of this problem is asset/liability matching. A firm has a liability stream over T time periods. It wants to fund these liabilities through investments. It does not wish to be restricted to regular bonds, but wants also to use bonds with uncertain cash flows, due to their higher returns.

The model used here (Zenios, 1991) assumes that only Mortgage-Backed Securities will be used. The model assumes that the firm begins with an existing portfolio of i bonds. The firm may sell and buy bonds only initially. It may change its portfolio initially (time period 0) by selling some bonds for cash, and then buying other bonds for cash. In future time periods, it may no longer buy or sell bonds. The model assumes that the firm will only get the cash flows from its bonds and the gain or loss of the value of the bond itself. Each scenario can be viewed as a flow graph, with a terminal state being the terminal wealth of the portfolio.

Although bonds may not be bought or sold in future time periods, at any given point in time, extra cash might be acquired from the bonds. In this model, the surplus may be invested for one year at the going one-year rate (forward rate). If the bonds themselves do not give enough to cover the liabilities in a given time t, borrowing is allowed.

We define a number of variables and constants below. Each will be preceded by either of the symbols C, DV or RV.
• C means that the value is a known constant.
• DV means that it is a decision variable that we are solving for.
• RV means that it is not known with certainty but only has a probability distribution associated with it.
• We deal with those by generating many scenarios, Monte Carlo style, and then including a separate constraint for each scenario:

C: Let in \_i = the value of bonds of type i in the original portfolio where in\_0 = the cash value in the original portfolio.
C: Let t = the current time period.
DV: Let x\_i = the amount of type i bond bought at t = 0. *These are variables for which we are solving.*
DV: Let y\_i = the amount of type i bond sold at t = 0. *These are variables for which we are solving.*
C: Let tsi = percentage transaction cost of selling asset i.
C: Let tbi = percentage transaction cost of buying asset i.
RV: Let w\_it = the cash flow generated by bond i at time t. This includes interest and principal and prepayment money if the bond is refinanced. Note that this is not a known cash flow. It is the cash flow assuming that the scenario that we are describing takes place.
RV: Let value\_it = the value of bond i in the portfolio at time t. This changes as time changes due to payment of principal and the possible prepayment option. This model by Zenios does not seem to take into account the change in the market value of the bond. This can be seen from constraint 2.
RV: Let p\_it = the cash flow of bond i at time t expressed as a percentage of the value of bond i (value\_it-1).
RV: Let lend\_t = the forward interest rate to lend at time t.
RV: Let lend\_t = surplus at time t invested until next period.
RV: Let borrow\_t = amount of cash owed at time t+1 due to borrowing at time t. Thus borrow\_t divided by a discount rate is borrowed at time t.
RV: Let borrow\_r = the forward interest rate to borrow at time t.
C: Let L\_t = liability due at time t.
Let PW = total portfolio wealth.

Using these variables, the following flow conservation constraints apply.

1. \( value\_i^0 + y\_i = (1 - tbi)x\_i + in\_i, \quad \forall i \text{ st } i > 0 \)
2. \( value\_it + w\_it = value\_it-1, \quad \forall t, \forall i \)
3. \( w\_it = p\_it value\_it-1, \quad \forall t, \forall i \)
4. \[ \sum_{i} w\_it + (1 + lend\_r\_t-1) lend\_t-1 + \frac{1}{(1 + borrow\_r\_t)} borrow\_t = borrow\_t-1 + lend\_t + L\_t, \quad \forall t \text{ st } t > 0 \]
5. \[ lend\_0 + \sum_{i} x\_i = \sum_{i} (1 - tsi)y\_i + \frac{1}{(1 + borrow\_r\_0)} borrow\_0 + in\_0 \]
Constraint 1 balances the original amount of bond \(i\) and the amount of bond \(i\) remaining after buying and selling. Constraint 5 deals with balancing the original cash amount with what is left after taking into account the buying and selling of all the bonds in the portfolio and the borrowing and lending of cash for the next period. Constraint 2 balances the value of each bond type before a time period with its value afterwards, taking cash outflow from the bonds into account. Constraint 3 simply allows \(w_i\) to be expressed as a percentage of the value of bond type \(i\). Constraint 4 balances the cash from one period to the next, taking borrowing, lending and bond cash flows into account. PW is computed by accumulating the total surplus cash and the market value of all leftover bonds minus any liabilities outstanding. This illustrates one scenario of interest rates.

As noted above regarding pricing an MBS, there are an unlimited number of potential scenarios that may be simulated. The flow conservation constraints apply to all scenarios. Note that the first “column” of nodes in the flow graph is at \(t=0\). This “column” represents the present: before any change in interest rates take place. Constraint 1 is deterministic; that is, it does not follow the interest rate random variable. Although constraint 5 applies to the cash in time period 0, it still relies on future periods through borrowing and lending, and is therefore still stochastic. A superscript \(s\) indicates that the scenario will be appended to those variables that differ in different scenarios. We then maximize the expected value of portfolio wealth over all scenarios subject to the constraints. Two more terms require definition.

Let \(s = \text{scenarios}\).
Let \(Pr(s) = \text{the probability of scenarios occurring}\).

Then we have the following Linear Program:

\[
\text{Max} \quad \sum_s Pr(s)PW
\]

\[s.t.\]

\[value_{i_0} + y_i = (1 - tb_i)x_i + in_i, \quad \forall i \quad st \quad i > 0\]

\[value_{i_1} + w_i = value_{i_{-1}}, \quad \forall t, \forall i, \forall s\]

\[w_{i_1} = p_ivalue_{i_{-1}}, \quad \forall t, \forall i, \forall s\]

\[\sum_{i_1} w_{i_1} + (1 + lend_{t_{-1}})lend_{t_{-1}} + \frac{1}{(1 + borrow_{t_{-1}})}borrow_{t_{-1}} = borrow_{t_{-1}} + lend_{t_{-1}} + L_t, \quad \forall t \quad st \quad t > 0, \forall s\]

\[lend_{0_1} + \sum_{i_1} x_i = \sum_{i_1} (1 - ts_i)y_i + \frac{1}{(1 + borrow_{0_{-1}})}borrow_{0_{-1}} + in_{0}, \quad \forall s\]

The formulation given above is known as a Stochastic Program. It is used because of the stochastic nature of the MBS.
Testing the foregoing on parallel computers has shown an ability to parallelize almost linearly by giving different scenarios to different processors.

6. More examples of Stochastic Linear programs

In the stochastic programming examples provided, only one random variable was necessary. The following are a few real-world applications of stochastic programming that model many assets with many random variables.

The following example is a very extensive application of stochastic programming that was modeled by the Frank Russell Company for Yasuda Kasai Insurance Company in Japan (Cariño, Kent et al., 1993, 1994, 1998). This model included many asset classes such as stocks and bonds, domestic and foreign. It even included loans made by the company to individuals. The inclusion of personal loans as a random variable shows the extent of the model; personal loans are not sold publicly and therefore must be separately evaluated.

The company was able to invest in an asset class in four ways: directly, through a specially created mutual fund, through ownership of a foreign subsidiary, or through lending to a foreign subsidiary. Furthermore, different insurance policies have different legal requirements which translate into different fund accounts for different policies. In this case, there were five insurance policies. Letting \( n \) be the number of asset classes in which the company invests, there are \( 4 \times n \times 5 \) allocation classes, that is, possible ways of allocating the funds. This would translate into \( 4 \times n \times 5 \) decision variables for the mathematical problem (columns). The numerous regulatory constraints and liability obligations would translate into constraints (rows) in the mathematical program.

Certain relationships must hold between the asset-income side of the company and the liability side. These relationships extend across time periods, and the model had to include many time periods. Thus there are numerous constraints linking one period to the next. What emanates from this is a very large linear program — even before taking scenarios for the stochastic parts of the problem.

The current mortgage debacle should serve as a reminder that asset allocation decisions must be made carefully and that we must resist the temptation to include seemingly safe securities without careful consideration.

As expected, many uncertainties exist on the asset-income side. These are the random variables associated with the asset classes being considered. Additionally, in this case, as in the case of many insurance companies, there exist uncertainties on the liability side. Every policy sold can be viewed as consisting of two components. One is the strict insurance side. This part of the policy pays for any future insurance claims that might be made against the company. The second component is for an investment by the policy holder. The insurance company guarantees a certain minimum return for each type of policy and it actually pays policy holders returns on their policies. In addition to the minimum return, each year an additional return is announced depending on market conditions in order to increase the company’s competitiveness by giving high returns to customers. These announced returns are the random variables on the liability side.

All these random variables are inputs to the linear program. Scenarios are generated for these random variables. Each of these is then fitted to the linear program, making the linear
program grow in proportion to the number of scenarios. It is the expected value of all these scenarios that is input to the objective function to be solved.

It should be noted that mathematical and stochastic programs, linear or otherwise, have been used to solve a myriad of problems in addition to those of cash flow/liability matching. As an example they have been applied to the labor input decisions of a typical rural household in an environment with risky agricultural technologies and off-farm employment opportunities (Becker, 1990).

Although some of the larger applications, such as the cash flow/liability matching discussed in this paper are recent, mathematical programs modeling uncertainty are not new. Reservoir systems (Houck et al, 1978), Planning models (Tintner and Raghavan, 1970) and other decisions necessary for multi-national firms (Salmi, 1974; Fourcans and Hindelang, 1974) have been modeled.

Recently there have been formulations of the bond portfolio management problem as a stochastic program. In fact, some of these recent studies have attempted to use parallel computers to help solve very large stochastic programs (Moriggia et al, 1998). There have also been attempts at developing general software to help users to design plans for achieving financial goals. One such software claims that it is actually used to optimize project portfolio performance in oil and gas applications and in capital allocation and budgeting for investments in technology (April et al, 2002).

Findings and Conclusion

In the recent and ongoing crisis, conservative investors — including banks, mutual funds and retirement funds — have invested heavily in securities without doing a proper and rigorous analysis. When these securities, originally seen as good investments, lost their value, many stable companies holding onto them were unable to meet their steady liability obligations, which in turn led to default on their obligations and bankruptcy.

The current mortgage debacle should serve as a reminder that asset allocation decisions must be made carefully and that we must resist the temptation to include seemingly safe securities without careful consideration.

Mathematical programming provides an extremely helpful tool in analysis.

In this article we have illustrated methods of modeling for the asset allocation problem. We then focused on mathematical programming, in particular two linear programs, to solve cash inflow/liability matching problems. These models are suited to the conservative investor who would like to take advantage of securities that offer decent returns, but who at the same time need to be sure that their obligations are met.

We also provided a number of real-world application examples, illustrating the applicability of these techniques to both past and current real-world applications.

We believe that mathematical programming is a robust method for appropriate companies such as insurance companies, mutual fund and retirement fund managers, banks, and others to help them meet their two primary objectives: to invest their assets wisely, and to satisfy their obligations in as riskless a manner as possible.
References


Strategic Analysis: Approaching Continuous Improvement Proactively

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Abstract

This paper examines the experience of introducing a continuous improvement tool to the management of an advance technology software engineering company. The model, Strategic Analysis, applies Appreciative Inquiry, an intervention that builds on organizational success, to the needs assessment process. The case highlights the potential of Strategic Analysis to encourage innovation.

Introduction

Surviving in today’s global business environment requires that organizations have the capability to respond efficiently and effectively to continuing and emerging economic and political challenges, even those that might be extremely difficult to anticipate. They must be flexible enough to make strategic changes in response to those challenges, with no discernible lapse in productivity (15, 28). Beyond survival, success in competition demands continuous improvement of both business performance and quality of product (8).

The continuous process improvement of an organization requires its leadership to understand its history of competitive successes and failures (31). There appears to be universal agreement that a thorough needs assessment should be an integral part of all continuous improvement plans and budgets, yet the literature also acknowledges that it often is not done in organizations (15).

Strategic Analysis, described below, provides the vision and direction that can promote organizational success (4). This is accomplished in two ways: by continually focusing the strategic plan and establishing the context for continuous process improvement and (8) by developing the organization’s enterprise portfolio, specifying the businesses that should be included and the level of performance required, the partnerships that are likely to be most profitable and the strategies by which organizations are most likely to achieve well-defined goals and objectives (10, 16, 17).

...assessment can identify and clarify the challenges facing an organization and the policy options available to the organization’s leadership to confront these challenges.

Needs Assessment

Needs assessment is a process of identifying deficiencies between current and desired results (30). The process systematically attempts to identify gaps between needs and capabilities. The identified needs are prioritized based on such criteria as the cost of eliminating the need against the cost for ignoring it, and selecting the most important needs (problems or opportunities) for reduction or elimination (18). A needs assessment “evaluates whether there is a ‘case for action’ or a ‘business case’ for modifying, enhancing, or replacing the
current system” (19, p. 13). Exhibit 1 below outlines the steps in a needs assessment.

Exhibit 1: Needs Analysis Model

1. Identify performance standards
2. Assess current performance
3. Identify gaps
4. Prioritize needs
5. Develop strategies to close gaps

There are a number of reasons to undertake a needs assessment. The assessment can identify and clarify the challenges facing an organization, and the policy options available to the organization’s leadership to confront these challenges. When a performance problem has emerged, the first step is to determine its exact nature. Next, data suggesting ideal performance standards are collected. From those data, the current performance can be compared to the ideal performance standards, revealing the gaps between the two. Once the causes of these gaps have been determined, appropriate strategies for closing those gaps can be identified and prioritized (19).

Appreciative Inquiry

By contrast, Appreciative Inquiry is more of a sufficiency model, focusing on what the organization is doing satisfactorily. Appreciative Inquiry uses positive dialogue as a means of uncovering stories of organizational successes (5). The process then builds upon those stories to promote future success. The model was developed in response to action research, which concentrates on finding organizational problems in need of solutions, and then proposing the development of an action plan to correct the problems that are identified (12).

Appreciative Inquiry posits that people construct reality through their social interactions, as opposed to the positivist paradigm that states social knowledge is the result of objective observation (6). The model is based on the premise that there is something already present in every organization that can be leveraged to help achieve its goals (14).

Critics of Appreciative Inquiry point out that the process is too focused on the positive, often at the expense of the organization’s shortcomings (9, 11, 23) Perhaps most telling of all, Rogers and Fraser (26) argue that Appreciative Inquiry is ill suited to identifying underlying organizational problems, which can be ignored by focusing solely on building organizational strengths. Exhibit 2 shows the Appreciative Inquiry process.

Exhibit 2: Appreciative Inquiry Model

Discover
- Strategic Context
- Positive Core

Deliver
- Structure
- Implement

Dream
- Purpose
- Vision

Design
- Relationships & Organization
The Strategic Analysis Model

Today’s highly competitive international business environment requires that organizational strategies must be deliberate and proactive (1). Anderson argues that such strategies “must be perceived as a proactive process which anticipates trends and future changes and which prepares people to meet them” (1, p.23) Needs can be identified reactively by responding to a problem, or proactively by identifying current and future needs before they develop into problems (25). Therefore, while needs assessment is a critical process to determine the cause of a problem before making a decision, it is by definition a reactive process.

The Strategic Analysis Model is a proactive continuous improvement model that combines elements of needs assessment with Appreciative Inquiry. By combining needs assessment models and Appreciative Inquiry, the Strategic Analysis model takes advantage of the positive aspects of each model and redresses shortcomings of each. In so doing, the organization can become more anticipatory, resulting in a more proactive approach to continuous process improvement. Continuous improvement entails constant enhancement of customer satisfaction, facilitated by increasing organizational efficiency and effectiveness. Strategic Analysis is a diagnostic tool that identifies those aspects of organizational behavior most in need of continuous improvement (22).

In uncertain environments, Strategic Analysis is able to provide organizations with a clear framework without getting lost in the chaos of change. This balancing act of utilizing past experiences, solving immediate problems, and preparing for the future by detecting patterns or threats requires the ability to manage strategic change (19). Leaders committed to considering a wide variety of alternative strategies for change, including those that are unconventional, have improved their chances for success (15, 19). It is therefore important to recognize that the incorporation of new technology is just a single component of an overall strategy in addressing continuous improvement.

Appreciative Evaluation

Every organization requires a valid and reliable means of evaluating the extent to which it is meeting its specified goals (27). The evaluation must be appropriate to the context in which it is conducted. Because its focus is on positive organizational outcomes, Appreciative Evaluation was selected as the most appropriate tool for evaluating the Strategic Analysis process (24) and was included at each phase.

Closely related to Appreciative Inquiry, described earlier, Appreciative Evaluation identifies what is best about an organization (21). It is a tool used to discover what animates the organization and what it needs more of. An evaluation approach that focuses on what is working, identifies a point of departure from which to launch efforts at organizational improvement and to move the organization

Exhibit 3: Strategic Analysis Model

Discover: organizational visioning and stakeholder analysis

Dream: generate ideas and concepts for moving the organization forward

Design: create initiatives minimizing limitations

Develop: strategies to realize initiatives and meet organizational needs

Distill: refine and prioritize initiatives

Deliver: implement strategies for continuous improvement

Appreciative Evaluation
forward in a positive direction. The goal is to leverage the organization’s strengths in order to realize the goals envisioned by its participants and stakeholders (20).

...there is something already present in every organization that can be leveraged to help achieve its goals.

During each phase of the process, questions designed to uncover what is best in the current system are developed. Answers to these questions provide an understanding of the role each factor plays in giving life to the organization. Each question is stated affirmatively (7). Appreciative Evaluation regards those who benefit as key actors in the process. They are not seen as subjects or objects of study, but rather as participants who are involved in understanding the process, assessing what was successful and proposing solutions to overcome obstacles and challenges (13). Evaluation, then, is seen not as an activity external to the process but rather as an integral component of that process.

The evaluation of Strategic Analysis involves assessing the extent to which the continuous improvement process meets the needs and goals of all participants and stakeholders (2). The case of Visualization Inc, below, is presented and analyzed to demonstrate the application of the Strategic Analysis model. A case study approach uses a particular case to gain insight into an issue or theory. In this type of investigation, the details of the particular case being examined are important only insofar as they serve to illustrate the viability of the process being investigated, which, in this instance, is enhancing organizational efficiency and effectiveness (29). All company and individual names have been changed to preserve the privacy and confidentiality of those involved.

Case Example:
Visualization Inc. and the application of Strategic Analysis model

Based in a large city, Visualization Inc. was a small start-up software engineering technology company. The company was founded in 2000 by five engineering graduates who comprised the management team. Martin Rausch, CEO, a mechanical engineer, managed the company’s overall operations. CTO Venetria Johnson led the product development team. She had been the key software architect for all product releases. Jamie Cypher, Director of Strategic Planning, was responsible for the overall operational strategy. He managed project teams, managed the company’s business and financial plans and conducted the annual performance reviews. As Visualization Inc.’s technical advisor, Janet Maddox provided direction to the company on future products. Rounding out the management team was John Fang, Director of Development, who had the responsibility of attracting investments and writing grant proposals to support their research and development efforts. The company employed 44 people.

Visualization Inc. began with one software tool: ImageSeek, a comprehensive resource for industrial information, products, services and CAD drawings. ImageSeek permitted manufacturing suppliers and distributors to connect with their partners and suppliers using shape matching technology and a product/service relevance approach. Using their proprietary shape matching technology and product/service relevance algorithm, they were able to deliver information about products and services from small and medium sized companies to hundreds of thousands of potential customers within days, changing the way manufacturers were able to market and distribute their products and services.
As director of Strategic Planning, Jamie Cypher recognized the need for a vehicle through which the company could continuously improve its products and services and persuaded the management team of the importance of engaging in such a process routinely. To help ensure that their in-house talent pool continued to grow and develop, Visualization Inc.’s strategic goals and objectives included a commitment not to outsource any of the company’s work. The authors had developed the Strategic Analysis model, which met Visualization Inc.’s needs for a model that would align with their strategic goals and objectives and would cause the company to always ‘think improvement.’ The model would guide Visualization Inc.’s efforts to improve its business performance.

Case Analysis

Preconditions: For Strategic Analysis to succeed, preconditions in terms of company support and resources had to be met within the organization. In the case of Visualization Inc., the concept of continuous improvement was embedded into the company’s DNA. As an innovative organization, they understood that they needed to stay competitive. To implement Strategic Analysis effectively, they incorporated the model into the company’s strategic plan. The organizational strategic plan is an integral aspect of the Strategic Analysis Model.

An understanding of the organization’s strategic direction is crucial to the implementation of the model. The strategic plan incorporates information about the external and internal environment, which includes customers, markets, human and financial resources and technology. To accomplish this, the senior management, which included Martin Rausch, CEO, CTO Venetria Johnson, Jamie Cypher, Director of Strategic Planning, and John Fang, Director of Development, formed a Strategic Analysis team that had the responsibility of moving the continuous improvement plan forward. The team consisted of senior management and four junior employee representatives, one from each department. During senior management’s monthly meetings a portion of the time was set aside for Strategic Analysis. The junior members – the software development team— were invited to attend the Strategic Analysis portion of each meeting. The software development team consisted of the project manager, requirements analyst, system architect, software tester and the support technician.

Phase 1: Discovery. In the first step of the Strategic Analysis Model, the Discovery phase, opportunities for continuous improvement building on the organization’s past successes were identified. This involved conducting a situation analysis, which consisted of assessing the current conditions, operations, processes and work environment. Since needs analyses are undertaken to help decide whether the current system is satisfactory, a formal vision statement set the expectations for system performance in both the short and long term (19).

This phase involved three main tasks: (1) organizational visioning, (2) stakeholder analysis and (3) evaluation of the current system. Organizational visioning entailed articulating a vision for the ideal enterprise system. Visualization Inc. had been extremely successful working with manufacturers, suppliers and vendors of ImageSeek, but needed to meet the requirements of their customers. In anticipation of future customer needs, Visualization Inc. sought ways to improve their services to customers.

During the discovery phase of the Strategic Analysis Model, the management team determined that customers may become dissatisfied with some aspects of ImageSeek. In this phase, a conscious decision was made to limit the inquiry to what was positive, rather than to analyze what was deficient. The focus was on the causes of success and building upon those successes as the organization moves into the future.
During this phase, the Appreciative Evaluation process involved asking the question “What has been our major discovery?” To respond to this question, Visualization Inc collected data from customers that indicated they had concerns about some aspects of ImageSeek. The team determined that their current software had some limitations. Having determined that the focus of the continuous improvement would be on software development, CTO Venetria Johnson emerged as the most appropriate team leader. The identification of the team leader enabled the team to move to the Dream stage in the model.

Phase II: Dream. This was the ideation phase where ideas were formed and related. This phase involved identifying and researching alternative solutions. At this point, the software development team took over most of the process. There were also many alternatives to consider with respect to the current system. Common themes and patterns were identified to construct a vision of an improved organization. The organization was re-created by developing “provocative propositions,” to merge the best of what has been into visions of what the organization could become, grounded in the reality of the organization’s experiences.

The team generated a list of criteria that ImageSeek needed to address. They wanted a program that promoted better program planning, allowing project managers to have at their fingertips parts/components available in the company’s internal repository as well as across the supply-chain. The software program should also enable sourcing and procurement personnel to identify similar parts for easy categorization and consolidated sourcing, even if part numbers are different. In so doing, companies using this software should be able to lower their sourcing costs and overall production costs.

Before moving on to the next phase, the team evaluated their efforts during the Dream phase. The Appreciative Evaluation question for this phase was “What challenges might come our way and how might we meet them?” (32). By asking this question, the team was better able to anticipate any problems that might arise.

Phase III: Design. Once the list of criteria that ImageSeek needed to address was developed, the project was assigned to the software design team. They began by evaluating potential solutions to support the continuous improvement process. The list of criteria gathered in the Dream phase was used by the designer to create prototypes. Input from stakeholders, including customers and technical staff, was involved throughout the process to ensure that all their requirements were incorporated into the new design.

Recognizing the need to provide a smooth transition for current users, the elimination and refinement of alternatives led to a number of options. These varied from actions that marginally enhanced the current system to replacing it entirely. Each option clearly specified a course of action, estimated costs, outlined benefits and listed pros and cons (19). The first option was the least financially risky and technologically aggressive. This entailed continuing with the current levels of software development investment and not funding major software development efforts. The second option was moderately economically aggressive and would have been a riskier move for the company. However, the dividends from this investment, if realized, would be enormously beneficial to both the company and the customers. The causes of the software problems of the current system would need to be identified and suggested options and prototypes developed to resolve them. This would require a substantial outlay of capital. The team decided to go with the second option, because while it was financially
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risky, it was not as risky as the third option, which required a much greater financial outlay to completely revamp the software. While there were no guarantees in terms of financial dividends, a totally revised system would ensure that there would be fewer problems with the software in the future and would allow for much easier updates.

This balancing act of utilizing past experiences, solving immediate problems and preparing for the future by detecting patterns or threats requires the ability to manage strategic change.

Proposed changes to the organization, and the new ideas created to support those changes, were evaluated by the extent to which they moved the organization in the direction of the ideal they envisioned in the Dream stage. Appreciative Evaluation for this phase involved asking the following questions: “What will it take to create change? What needs our immediate attention going forward?” By addressing these questions, the team was able to determine the extent to which they would be able to implement the changes.

Phase IV: Distill: During this phase, initiatives, including prototypes generated during the design phase, were refined and prioritized. The design documents underwent various stages of reviews and approvals, before moving to support the Development Phase 4. The alternatives were evaluated, considering the specification of the benefits, costs and risks of each alternative and beta tested to determine feasibility. For example, replacing the current system by outsourcing the information technology staff supporting it was not an option for the company because that would have been in direct contradiction with their strategic objectives and values. It would have been futile for them to research the ranges for outsourcing costs.

To evaluate this phase, the team asked the questions: “How can we support each other in taking the next steps?” “What unique contribution can we each make?” The team was able to recognize each other’s efforts and take advantage of each member’s strengths in the project. For example, the requirements analyst focused on what the end users would need to be more effective at their jobs. The system architect had the task of putting together the skeleton of the software project. Each decision that the system architect made was carefully considered, because a wrong move at the beginning of a project can have damaging effects later in the software development life cycle. The software tester was responsible for debugging the software and ensuring that the software worked properly and the support technician raised any technical support issues. The project manager worked with management to ensure that they provided the resources and support required, as well as dealing with team issues that are negatively impacting a team’s productivity.

Phase V: Develop. During this phase, initiatives from the development phase were refined and prioritized. The experiences, analyses and judgments of stakeholders were used to develop a list of options and recommendations from all of the alternatives that previous steps had identified. While acknowledging that software security is a serious problem and, if present trends continue, could be much worse in the future, the team offered recommendations for improving the software development process. The team recommended that the software include security features that would protect it from any breaches. The quality of software produced by the industry is extremely variable and there is inadequate oversight in some critical areas. It was important for the team that the software they developed was reliable, with the potential to bring dramatic benefits to end-users, such as linking the product to cost.
The main criteria for an alternative to be retained as an option were improvement over the status quo and the ability and willingness to implement that course of action (19). This stage of the process specifically focused on plans and actions at both the personal and organizational levels. At this stage the organization finalized the decision and took action to decide on a course of action.

The organization committed to the innovations identified in the Design phase by developing strategies based upon specific organizational needs.

As a result of the process, another software program was developed. ImageCost was developed as a next generation engineering advisory system (EAS). It analyzed CAD models or design criteria that had been manually entered and recommended the optimal process for manufacturing the part. ImageCost provided seamless integration with major CAD systems. ImageCost provided detailed geometric data extraction directly from the CAD file such as weight, volume, holes, thickness distribution and other relevant features. It also gave cost estimates to compare with supplier requests for quotations (RFQs). It regularly updated the knowledge base to incorporate the latest manufacturing technology.

To evaluate this phase, the team members asked the question: “What opportunities do we see in this situation?” As a result of the successful development of ImageCost, Visualization Inc. embarked on other development projects, including a program that integrates engineering and design functions into the entire product lifecycle — from sales to final manufacturing and assembly.

**Phase VI: Delivery:** Once a course of action was chosen, the decision favored the implementation of change. The next step was to prepare the organization for the change. The decision led to steps to establish a strategy for change whereby the culture of the organization and its readiness for change were determined (19). After two years of development, ImageCost was delivered to customers. The successful development and delivery of ImageCost was a direct result of the application of the Strategic Analysis. During the final phase, the evaluation focused on positive lessons learned and how the continuous improvement process can be sustained.

**Organizational Outcomes of Strategic Analysis**

By conducting a technology investment evaluation, (19), Visualization Inc. was able to determine the impact of Strategic Analysis on their company.

As shown in Exhibit 4, T1 represents what the result of an aggressive technology investment would have been in terms of profits. However, it was a much riskier strategy than the other two. T2 represents the result of a moderate technology investment. While the profits from this technology investment are not as high, less risk is incurred. T3, on the other hand, represents the result of maintaining the current investment in ImageSeek and not seeking further research and development.
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opportunities. The break even analysis results were used for decision making as to whether or not to abandon the project if forecasts show that below break even values are likely to occur (3). Through Strategic Analysis, the company determined that their best course of action was T2.

Strategic Analysis can be a vehicle for encouraging consensus by identifying the benefits, costs and risks of different courses of action.

A successful operational capability demonstration (OCD) was performed prior to full implementation and was highly successful. Interfaces with all system components were fully functional and seamless to the end-users. The software developed was fully compatible with and integrated into the existing LAN and software suite.

Conclusion

Strategic Analysis allows for the creation of numerous advantages for the organization.

- First, it allows the organization to focus on the positive and build on its successes.
- Second, while focusing on the positive, it recognizes shortcomings, enabling the organization to anticipate and prepare a planned and coordinated response when adverse factors such as competition threaten the organization’s well being.
- Third, it allows the organization to identify new technologies, methods or approaches that will increase its competitive advantage by increasing effectiveness and efficiency, thereby reducing costs.
- Finally, it allows the organization to anticipate problems, thereby reducing the chances of unpleasant surprises, adverse consequences and calamities.

The Strategic Analysis process is especially valuable to organizations that lack consensus about how and when to proceed with continuous process improvement. Continuous improvement is an on-going process that has lasting impacts on operating processes and personnel. Strategic Analysis can be a vehicle for encouraging consensus by identifying the benefits, costs and risks of different courses of action. By documenting and sometimes quantifying system benefits, benchmarks are set (e.g., days to close, hours to process payroll, days for purchase order approval, reduced cycle time for budget development, etc.) that can be used throughout the implementation process to gauge whether the business drivers for the investment in new systems are likely to be realized (19). By virtue of its focus on continuous improvement and evaluation, Strategic Analysis can position companies for success in this highly competitive global environment.
References


Assessing Redundancy’s Impact on the Reliability of Microcontroller/Processor-Based Systems in Mission Critical Applications

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Abstract

Many commercial and governmental systems employing microcontrollers or microprocessors require very high mission reliability. This goal can be achieved by incorporating single units with exceptionally low failure rates that are correspondingly costly, or under certain circumstances, by constructing systems employing redundant inferior units that are relatively cheap. This paper’s objective is to provide an aid to cost-effective design by analyzing and interpreting the absolute and relative mission reliability of such systems as a function of redundancy level for linked, parallel units operating in either inactive or active standby modes under constant hazard rates.

Problem Statement and Computational Objectives

A critical design consideration in the ubiquitous usage of microcontrollers/processors in embedded control and other systems on which commercial and governmental organizations depend so heavily is high mission reliability. Such systems include, for example, remote sensing/surveillance in security applications, robotics, transportation, health care, process control, file servers, etc. The term “mission reliability” denotes the likelihood that for some mission length, T, such systems will perform under stated conditions within design specifications (Blanchard). Two non-mutually exclusive approaches to enhancing a system’s reliability involve the improvement of individual component reliability and the incorporation of redundancy. Within these two approaches are two further options: redundant elements operating in either inactive or active standby.

As one might expect, because of the increasing application of embedded microcontrollers in civil and military systems where reliability is crucial (e.g., the automatic braking and stability control systems in cars, pacemakers, flight control systems, etc.), an extensive literature has evolved on this subject — some in the open literature [ARIC], some proprietary and some classified. The world’s largest technical society, the Institute for Electrical and Electronic Engineers, even has a separate society devoted to this subject. As noted in the following, the two alternative modes of operation (inactive standby and active redundant units) have been addressed in the literature along with their relevant equations.

The objective of this paper is to evaluate the cost effectiveness of systems design by analyzing their absolute and relative mission reliabilities. The main departure of this paper is to provide specific quantitative results that reveal the incremental, relative gains (if any) in reliability versus the level of redundancy. This is done for a general system configuration with either inactive or active redundant components that operate under various failure rate scenarios — 144 in all — that don’t appear to have been considered explicitly elsewhere. The parametric results provided in six accompanying exhibits are fully interpreted in
For a mission’s success, the unit must succeed in operating within specifications and under stated conditions until time $T$.

For Case (2), each redundant “path” of the system, as well as the primary path, is assumed to consist of independent parallel-series arrangements of independent units and independent, switched power supplies. All are considered in inactive, standby mode until the controller detects a failure of the primary unit and activates a redundant path and so on. During inactivity, it is assumed that neither a switched power source nor the unit coupled in series with it can fail.

In both Cases (2) and (3), failures are assumed to be independent, with exponential failure probability densities (i.e., constant hazard rate).
configuration for a mission of duration \( T \) can be readily determined (ARINC) as the following:

\[
R(T, N) = e^{-\left(r + r_p + r_c\right) N} \prod_{i=0}^{N-1} (1 / i !)(r + r_p) T
\]

In this equation, \( r, r_p, \) and \( r_c \) are the respective constant failure rates of the individual, independent units, switched power supplies and single supervisory controller. From the preceding equation for a single unit with no redundant elements and with some prescribed reliability, \( R(T,1) \), \( T \) can be found as:

\[
T = -\ln[R_1(T)] = -\ln[R(T,1)] = -\ln[e^{-r T}] / r
\]

This allows the values of \( R(T,N) \) to be computed for different overall mission reliabilities and various ratios of the three failure rate parameters, including the case where all failure rates are equal. The improvements or degradations in percent relative reliabilities, \( 100* R(T,N)/ R(T,1) \) are summarized in the accompanying Exhibits (1-5) as a function of redundancy level for single unit mission reliabilities of 90% and 80%.

**Redundant Active Units**

This case also incorporates redundancy to improve overall mission reliability. Here, however, each redundant path of the system, as well as the primary path, is assumed to consist of parallel arrangements of \( N \) identical active units (with some internal controller for handling responses to service requests and perhaps functioning to determine load sharing). Since all units are active, there is no accompanying supervisory controller (monitor or detector) to switch separate power supplies to activate the standby (inactive) redundant units as in the previous case. As discussed earlier, failures of the \( N \) active units are assumed to be independent with exponential failure densities (i.e., constant hazard rate). Employing the general theorem for the joint probability of independent composite events (Hoel), the reliability of the redundant unit configuration for a mission of duration \( T \) can be readily determined as the following:

\[
R(T, N) = 1 - \left[1 - e^{-r T}\right]^N \quad T \geq 0
\]

where \( r_i \) is the failure rate of the \( i \)th unit. If these are all equal to \( r \), the above reliability equation becomes the familiar expression:

\[
R(T, N) = 1 - \left[1 - e^{-r T}\right]^N \quad T \geq 0
\]

For \( N=1 \), this result devolves to the earlier equation for a single primary active unit with no redundancy. For a prescribed mission reliability, this and the result for \( R(T,1) \) leads to the outcomes given in the accompanying Exhibit 6 for the percentage relative reliability improvements (or degradations) as a function of active redundancy level, \( N>1 \).

**Findings**

Both dynamic (i.e., inactive standby units and active redundant units) provide noteworthy insights regarding the impact of redundancy on overall system mission reliability. The foregoing reliability equations and resultant summary exhibits of relative reliability versus redundancy level and relative component failure rates lead to the following conclusions, which can provide any system designer with useful, quantitative guidelines:

**Inactive Parallel Units:**

(1) For this dynamic failure model (with exponential failure probability density function), Exhibit 1 shows that the effects of redundancy are quite varied and not always beneficial, depending on the single unit reliability required and the associated common failure rate of the key independent system components: supervisory controller/monitor, switched power supplies and microcontroller/processors (“units”).
Exhibit 1
Inactive Standby Redundant Units
% Relative Reliability vs. Redundancy Level
Equal Component Failure Rates

<table>
<thead>
<tr>
<th>Redundancy Level</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100.00</td>
<td>99.98</td>
<td>99.99</td>
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<tr>
<td></td>
<td>100.00</td>
<td>98.07</td>
<td>99.87</td>
<td>98.94</td>
</tr>
<tr>
<td></td>
<td>92.56</td>
<td>98.94</td>
<td>96.42</td>
<td>99.39</td>
</tr>
<tr>
<td></td>
<td>83.95</td>
<td>99.39</td>
<td>99.99</td>
<td>99.99</td>
</tr>
</tbody>
</table>

(2) Exhibit 1 indicates that when all the system elements have identical exponential failure distributions, the relative reliability of the redundant system (i.e., relative to a non-redundant, primary system unit) is below 100% and is limited by the failure rate of the supervisory controller. The relative reliability is worst for just one redundant path, but thereafter rises monotonically and rapidly with the level of redundancy. There is practically no difference after the third redundant path is added. Thus, with equal component failure rates, redundancy is useless and wasteful.

(3) Exhibits 2–3 reveal that when the supervisory controller has a much better failure rate than that of the system units, the effects of redundancy are all positive for all ratios of switched power supply to computer failure rate at or below 1:1. For a required mission reliability of 90%, the system’s relative reliability quickly approaches the asymptotic value of 111% (i.e., compared to a non-redundant system) as the redundancy level rises – achieving this with about 3 redundant paths. Incrementally, the greatest improvement comes about with just one redundant path.

For a required mission reliability of 80%, Exhibit 3 shows essentially the same trends with redundancy level, but with more significant impacts. The incremental gain of one redundant path is larger than that for 90% mission reliability and the asymptotic relative reliability is also higher at about 125%.

Exhibit 2
Inactive Standby Redundant Units
% Relative Reliability vs. Redundancy Level
and Switched Power Supply Failure Rate

<table>
<thead>
<tr>
<th>Supervisory Controller Failure Rate = 1% Unit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Unit Required Reliability for Period, T: 90%</td>
</tr>
<tr>
<td>Ratio of Switched Power Supply to Unit Failure Rate</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
</tr>
</tbody>
</table>

Exhibit 3
Inactive Standby Redundant Units
% Relative Reliability vs. Redundancy Level
and Switched Power Supply Failure Rate

<table>
<thead>
<tr>
<th>Supervisory Controller Failure Rate = 1% Unit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Unit Required Reliability for Period, T: 80%</td>
</tr>
<tr>
<td>Ratio of Switched Power Supply to Unit Failure Rate</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>3</td>
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<td>5</td>
</tr>
</tbody>
</table>
Exhibit 4 indicates that when each of the switched power supplies has a much better failure rate than that of the system units, the effects of redundancy are essentially all positive for all ratios of supervisory controller to unit failure rate below 1:1. Again, the largest incremental gain is with one redundant path. For a required mission reliability of 90%, the system relative reliability rapidly approaches different asymptotic values, depending on the ratio of monitor-to-unit failure rates.

As the monitor gets more reliable, the relative reliability rises to its asymptote of 111% by inclusion of around two redundant paths.

Clearly, for this or any other probability density function, a unit’s reliability must eventually approach zero; i.e., the unit will wear out ultimately or fail due to some destructive event.

Exhibit 4
Inactive Standby Redundant Units
% Relative Reliability vs. Redundancy Level and Supervisory Controller Failure Rate

Switched Power Supply Failure Rate = 1% Unit Rate
Single Unit Required Reliability for Period, T: 90%

Ratio of Switched Power Supply to Unit Failure Rate
1 0.5 0.1 0.01

Redundancy Level
0 100.00 100.00 100.00 100.00
1 99.47 104.85 109.37 110.41
2 99.98 105.39 109.93 110.97
3 100.00 105.41 109.95 110.99
4 100.00 105.41 109.95 110.99
5 100.00 105.41 109.95 110.99

Exhibit 5, which provides the outcomes corresponding to a mission reliability of 80%, evidences similar, but stronger results.

Exhibit 5
Inactive Standby Redundant Units
% Relative Reliability vs. Redundancy Level and Supervisory Controller Failure Rate

Switched Power Supply Failure Rate = 1% Unit Rate
Single Unit Required Reliability for Period, T: 80%

Ratio of Switched Power Supply to Unit Failure Rate
1 0.5 0.1 0.01

Redundancy Level
0 100.00 100.00 100.00 100.00
1 97.81 109.36 119.57 121.99
2 99.84 111.62 122.04 124.52
3 99.99 111.79 122.23 124.71
4 100.00 111.80 122.24 124.72
5 100.00 111.80 122.24 124.72

Active Parallel Units:

When the parallel units are active, there is no need for a supervisory controller (monitor) to activate any redundant unit that would be in standby, as in the previous model. Accordingly, Exhibit 6 reveals the following:

(1) Since the overall reliability functions start at 1 for T=0 and asymptotically approach 0 for large T, redundancy results in only marginal differences for short or long mission time, T.

(2) If all the units have identical failure rates, r, then the best incremental improvement in the redundant system’s reliability and mean time to failure comes with just one redundant unit, where r is the common failure rate. With N parallel paths in the system, the overall mean time to failure becomes N/r.
For all the single unit mission reliabilities examined (70%, 80%, 90%, 99%), asymptotic relative performance improvement is reached very quickly, with redundancy levels of just 1 or 2 units (especially for the higher mission probabilities).

Both dynamic models (i.e., inactive standby units and active redundant units) provide noteworthy insights regarding the impact of redundancy on overall system mission reliability. Of particular benefit to the reader is the possibly counterintuitive outcome that redundancy doesn’t always improve reliability.

### Exhibit 6
**Active Redundant Units**

<table>
<thead>
<tr>
<th>% Relative Reliability vs. Redundancy Level</th>
<th>Equal Component Failure Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Unit Required Reliability for Period, T:</td>
<td>99%  90%  80%  70%</td>
</tr>
<tr>
<td>Redundancy Level</td>
<td>0</td>
</tr>
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<td></td>
<td>1</td>
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<td>2</td>
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</tbody>
</table>

### Conclusion

This paper’s purpose has been to provide key insights on the way redundancy may enhance the performance of microcontroller/processor-based systems that require high reliability. The 144 practical scenarios that have been considered should be of value in achieving more cost-effective designs. As an important adjunct to the corresponding equations and numerous quantitative results, each of the findings has been fully interpreted. Of particular benefit to the reader is the possibly counterintuitive outcome that redundancy doesn’t always improve reliability. Furthermore, even when redundancy heightens reliability, the accompanying calculations reveal that the gains can be quite marginal, leading to wasteful designs.

### References


Problems with the Application of a Metric for CRA Compliance in Banking

Andrew Russakoff, The Tobin College of Business, St. John’s University

Abstract
This paper discusses the difficulties of applying the Community Reinvestment Act to banks. It finds that the complications of community investment needs and the opacity of compliance formulas make any banking act too small a commitment to have real effects.

Introduction
The objective of this paper is to suggest some of the complexities of economic investment with respect to the Community Reinvestment Act (CRA) that make the algebraic formulas of compliance far out of range of true measurement. I will discuss some of the ways that the government has attempted to implement CRA compliance, what this reveals about the implicit model for investment and how the metric for CRA compliance might be modified to better address the issue.

To address the issue of under-investment and underdevelopment in locations in need of a business stimulus, in 1977 Congress passed the Community Reinvestment Act (CRA). The law appoints overseers of the CRA interest (mostly the Federal Reserve Banks, but in some cases the Comptroller of the Currency) and rewards successes. Different minorities have very different issues in finding funding for entrepreneurial efforts. Inner city residents, who have been generations removed from individual ownership, respond differently than new immigrant Asians, who may have family or community resources upon which to draw. Of course, the law makes none of these distinctions. Instead, the law asks the banks to do better, however they see fit. In addition to the difficulty this creates, the law subtly encourages a cynical attitude to the process. The rewards follow whatever standards are created, not the ultimate social effect, because the time line for the rewards is so near. The results are calculated each year.

The CRA also overlooks the deeper analysis of community needs. If a Native American community needs infrastructure creation far more than it needs cash for some consumer goods, the CRA has no eyes to see it. This, unfortunately, is the most critically important point of this paper. If the complexity is too large and the measurement too arcane, then the law promises what it cannot deliver and leads to frustration all around — for politicians, for regulators, for bankers and for citizens looking for economic improvement in their neighborhoods.

If the CRA is given a hopeless task, we can at least see what it does and how. It is overseen by the Federal Reserve Bank system, which is charged with measuring and enforcing compliance. Most banks are regulated by this system, a small minority by other agencies. Because the standards must fit a complex and changing situation, the different Federal Reserve Banks have had to use different standards to judge compliance.

The target of CRA compliance is investment in low and moderate income areas. Because these differ in so many ways across the country, it is impossible to fix a single standard. The CRA does not even try. It lets the banks determine how investment is made and measures the success, or failure, solely by the dollars invested.
Implementation

The implementation of CRA standards is contingent upon two factors. First, there is the matter of standards — how are banks rated. Second, there is the matter of inducement — what are the rewards for a higher rating.

CRA standards are simple categories. Either a bank is satisfactory, or close, or poor, or bad. This already creates some computational difficulties. If a bank is judged to be “satisfactory,” that rating is the result of consideration of forms and presentation. There is no simple measure that finely separates the categories. The failure of the standards occurs, in part, because the determining factors are the presentation, rather than the results. The Federal Reserve looks at information provided by the bank and determines what the CRA compliance was. If a bank is poorly rated, it will be advised to change its presentation. What were its outreach programs for minority owned businesses? How were they advertised? Were all these efforts sufficiently detailed in the reports to the Federal Reserve?

Inherent in this system is the problem of judging process, rather than results. If a bank approaches a college (as happened at St John’s University) and asks for advice about marketing banking services to minority owned businesses, this can serve as the centerpiece for a presentation of CRA compliance. On the other hand, this may not have an easy translation into investment. Actual investment follows not from the advertising but from the examination of business prospects. The officials responsible for the granting of loans are also responsible for making ‘good’ loans. If the banker and the prospect do not find the same language of confidence, no loan will be made. The presentation was a success, but the loan was not made (cf. Stern).

In this regard, two observations come to mind. First is the perception of the banker as rigid and unsympathetic. The second is that the sociology of this interaction is well-known. Typically, people become bank officers after following a career emphasizing caution and restraint (routinization). It is probable that successful minority business people are more likely to be those willing to take risks, to invest both cash and time before there is much certainty of success. Clearly there is no formula sophisticated enough to balance these somewhat contradictory styles and produce good results for both sides.

Investment Model

From this cursory look, it can be seen that there are many competing models for the economic process. On the one hand, there is the purely economic model. This sees a business as building from some entrepreneurial experience. A man works at a repair shop, a retail outlet a small manufacturer. He recognizes that there is great growth potential for this activity in this situation. He approaches a bank for funding to proceed. His representation is so convincing that he gets the loan. With the money, he sets up his own business or expands his business and is soon doing so well that he is not only repaying his loan, but thinking of further expansion.

The more political model begins with the same scenario, but in this case the bank cannot recognize the potential of the applicant because of sex, race, religion or economic status. The applicant is denied. The local economy is stagnant. The kick-start of CRA promotes a more sympathetic view of the applicant, so the financing process can proceed successfully.

The sociological model imagines that at least some minority communities do not have or encourage the individualistic, entrepreneurial attitudes; nor do they have the confidence to risk savings and time to create new small businesses.
All these models suppose that there are regions of low economic activity. All imagine that the engine to lift the community must be members of the community itself. All see the individual entrepreneur as the active element. It is not clear why this is so uncritically accepted, apart from the observation that it suits the political climate. The last model does not regard all people as fundamentally the same, but considers the differences of race, etc., negligible in the larger view of the marketplace.

Examination of Metric

In each of the steps to creating a metric we have run into the political dimension. This political dimension is bound up in the nature of the players. The Federal Reserve Bank is not in a position to fine-tune interest rates for each individual bank based on a finely-tuned CRA rating system. The only tool of the Fed is the granting of permission for bank expansions and mergers. This is a very blunt instrument for this purpose. It is not at all clear that it is enough of an inducement to change bank policy in some significant way (cf. Dahl, et. al.).

Similarly on the other side of the equation, the CRA rating is not a very finely regulated one. A bank that has developed an outreach program through the cooperation of a university can show this as evidence of CRA compliance, whether or not it produces new investment. Banks may argue that their particular circumstances make any more numeric assessment inappropriate. Some communities have financial resources outside the normal banking system (cooperative loan societies, for example); others have difficulty accepting the entrepreneurial system.

In particular, I want to revisit the measurement of Dahl, Evanoff and Spivey (for the Federal Reserve Bank) for detecting the sensitivity of bank mortgage loans to CRA downgrading. This is an important example because it may reveal the weakness of introducing measurement standards in a situation which straddles two very different sets of values: the economic and the political.

Note: I am calling the non-economic factors political because I believe that is both a convenient shorthand and a reasonably appropriate description. If a bank has been downgraded in its CRA compliance, but cannot see acceptable alternative policies, it can simply ignore CRA costs and benefits, or it can approach local politicians and explain how they are potential contributors and that the agency is inappropriately unhelpful. Government learns that it must resist political pressures, but that ultimately it cannot refuse them all.

The Federal Reserve looks at information provided by the bank and determines what the CRA compliance was. If a bank is poorly rated, it will be advised to change its presentation.

There are two equations (models) that Dahl et al. use to analyze the sensitivity of banks to CRA downgrading:

Treatment effects:

\[ \Delta TLOAN(i,t) = a + d_{1}CRA\_DG(i,t-1) + d_{2}ASSET(i,t-1) + d_{3}\Delta MARKET(i,t) + d_{4}ECAP(i,t-1) + d_{5}TMORTG/ASSET(i,t-1) + d_{6}FDIC(i,t-1) + d_{7}HOLD(i,t-1) + e(i,t) \]

First, let us examine the variables:

\[ \Delta TLOAN(i,t) \] denotes the change in CRA targeted loans, the ratio of low-income mortgage loans to assets for bank \( i \) in year \( t \). The implication is that the Federal Reserve is looking for a change in the ratio, rather than simply a change in volume (dollars).
The first explanatory variable, CRA_DG(i, t-1) is a binary variable equal to one for a downgraded bank in year t-1, zero otherwise. The authors seem to want the comparisons to apply for years t-2, t-1, then t for the year of the downgrade. It is not presumed that t=0 refers to the year that showed the poor CRA results and t=1 for the year after the downgrading. With the definition given, this asks the regression to evaluate some constant for the zero year (in addition to the regression constant) that is not present in the year after (or any subsequent year).

ASSET(i, t-1) is the log of bank assets. One might have thought this should be the difference in bank assets. If bank assets had markedly changed, this should be allowed to explain bank lending. As presented this is not the change, but the log. Typically the log is used to make linear factors of multiplied variables. That does not apply here. The log ASSET term must be a rough guide to how a small community bank will be treated differently from a large multinational (for example, Citibank). Larger banks seem to have better results in CRA ratings. They also have more to gain.

ΔMARKET(i, t) is the dollar change in the volume of low or moderate income mortgage loans for all financial institutions in the bank’s market normalized by lagged total mortgage lending where the Metropolitan Statistical Area is used. This is a wonderful variable. By matching a bank’s performance with other banks in that specific area, it precisely looks at performance in a competitive manner.

ECAP(i, t-1) is the lagged ratio of equity capital to assets. It allows for the particular bank’s financial condition. As noted in the paper, this consideration follows other research (Harrison, 1999).

TMORG/ASSET(i, t-1) it the lagged ratio of low income mortgage loans to total assets. It allows for the possibility that the bank has a target ratio to which it moves so that changes in the low-income loans will have a ‘correction factor.’

FDIC(i, t-1) is a wonderful variable, noting that the Federal Deposit Insurance Corporation has different standards than the other supervisory organizations (Federal Reserve or Office of the Comptroller of the Currency). This ‘correction factor’ follows the research of Thomas (1998).

HOLD(i, t-1) is a binary variable valued at 1 if the bank is owned by a holding company and 0 otherwise. Apart from the suspicion that a holding company may be more susceptible to Federal Reserve pressure, it is not clear how often this will be an important factor and it appears that affiliated banks have higher CRA ratings.

The other terms in the equation are a constant and an error term.

If this equation fits the data with the usual standards of predictability (high correlation, significant ANOVA and significant coefficients), we will have a useful metric by which to scale the effects of CRA downgrading.

It is, however, a bit worrying that there are three binary variables: CRA_DG, HOLD and FDIC, (and ASSET is essentially another). Although these can only be binary variables, nevertheless they affect the degrees of freedom, by, for example, only comparing FDIC regulated banks with other FDIC banks. Only MARKET is directly related to the district in which the bank operates. The other variables are aggregated parts of bank reports.

It is significant that the authors report that banks do not respond to the downgrading. Perhaps the explanation for this lies in a closer examination of the decision mechanism of the banks. A better approach might be that
the legislation is attempting to solve a social problem with a financial tool that is both inappropriate and ineffective.

It is simplistic to see money as the answer to stagnant economic development. There must be a climate and an opportunity for growth and investment. In this regard, mortgage lending is associated with economic development, but is not, in itself, the answer. How often have we read of money used to enrich people who then take the money out of the neighborhood it was meant to serve?

It is simplistic to see money as the answer to stagnant economic development. There must be a climate and an opportunity for growth and investment.

Similar considerations apply to the probit model.

Probit model:

\[
PR(\text{CRA\_DG}(i,t-1)) = a + d1\text{HOLD}(i,t-2) + d2(\text{LOAN/ASSET})(i,t-2) + d3\text{ASSET}(i,t-2) + d4\text{MARKET}(i,t-2) + d5\text{ROA}(i,t-2) + d6\text{ECAP}(i,t-2) + d7\text{TMORTG/ASSET}(i,t-2) + d8\text{FDIC}(i,t-2) + u(i,t)
\]

\(PR(\text{CRA\_DG})(i,t-1)\) is the probability of a CRA downgrade for bank \(i\) in year \(t-1\). What makes this a probit model is that the dependent variable is in \((0,1)\).

The other new variables are explained below.

\(\text{LOAN/ASSET}(i, t-2)\) the ratio of total loans to assets when that is significant for bank officers. In this variable is the notion that active banks (with higher CRA ratings) tend to have high loan-to-asset ratios (Bierman et al. (1994) and Gunther (1999). The implication is that higher CRA is from more activity (and presumably, risk) rather than especially ‘smart’ loans.

\(\text{ROA}(i, t-2)\) is the ratio of net income to total assets, testing the role of profitability on CRA downgrades. It has been found that banks with poor profitability have higher CRA ratings (Bierman et al. (1994)). Gunther (1999) has related results in the other direction, showing that losses may re-direct investment away from CRA goals.

This model speaks directly to the issue of predicting CRA downgrading, but still employs two binary variables (HOLD and FDIC).

We have examined the two functions that are suggested to measure CRA compliance in the Federal Reserve publication. Whether term by term or as a group, the functions strictly measure the applicable investment of the bank. The subtleties of the functions have most to do with the characteristics of the bank itself (assets, etc.). Thus the chief measure of CRA compliance is monetary. How much money was spent. There is no quantification of the degree to which the investment furthers social goals. There is no measure of the effectiveness of the investment in job creation, in community building. In some ways the functions reveal the CRA compliance to be a species of tax on the banks. If you pay this tax (i.e. make investments), then you are in line to get rewards.

This examination reveals a number of troubling considerations. First, the use of binary variables is necessary, but makes the analysis more difficult. Now the comparisons are thrown out of the MSA, or to a restricted set of local institutions. The deeper reason for CRA, in any case, is lost.

In the light of Bierman et al., Gunther and Harrison, it does not appear that Federal Reserve’s CRA rating is a sharp-edged enough tool to create the economic activity for which it was intended.
We will omit for now the introduction of different levels of technical complexity. There may be some reason to believe that there are interactive effects that are worthy of inclusion, but mostly these would involve new social variables rather than more financial precision.

*Algebra is no short cut for common sense.*

**Findings and a Discussion of Alternatives**

Term by term, the two models make good sense about trying to normalize changes from before and after the CRA ratings change. Two important considerations arise. First, the models show there is little significant effect. CRA ratings do not affect bank performance. Second, the models cannot hold onto such a slippery subject. In the wake of GLBA, mortgage lending is just one aspect of economic activity and may not be even the most significant one in measuring community economic health. It may have been used because it is associated with that and, primarily, because it is measurable. Furthermore, the probit style is both more appropriate, because it models the categorical aspect of measurement of CRA and of Fed rewards, but at the same time it is inherently insensitive.

The Federal Reserve is reaching for better standards. The Sunshine provision of CRA, together with the Federal Reserve guidelines (seven communication efforts) and specially designated staff, reveal that the regulatory agencies would like the banks to come up with standards to apply. This embodies a refreshing honesty about the difficulty of measuring the immeasurable, but will not solve the underlying problem of forcing together radically different systems (cf. Jackson).

If Congress wants to change poorly performing economic areas, it must have some notion about what the root causes of the problems are and address those root causes. It serves neither the banks nor the communities to suppose that tinkering with loan access will do the work for them.

A more direct approach to community revitalization might include the creation of (or placement of) institutions (agency, school, prison, ‘green’ resource management), improvement of infrastructure (roads, communications, airports), establishment of preference (tax abatements or benefits). In short, different strategies for different areas.

The effort to find an objective measure of CRA compliance relies directly and entirely, on measuring how much money was put out for investment in the under-served community.

In the same fashion, a more sophisticated approach is necessary to distinguish the different types of banks. Some banks are quite small ‘retail’ operations. They offer traditional banking services (savings, checking, loans). By contrast some have no retail operations (the late Bear Stearns was an investment bank). There is an extent to which the big multinationals (Citibank, Chase) do both. Some of the more aggressive expanding banks seek to become bigger players by building on the retail side (consider here Wachovia, Commerce, Washington Mutual). If the particular under-served community is in need of infrastructure, an investment bank might be the best provider. If the community is ready for the entrepreneurial take-off, the more retail side might be the best fit.

As attractive as a CRA metric function may be, the ultimate judge of the success of the CRA may be that the government must exercise the political will to differentiate different needs and different providing
institutions. Furthermore, community needs must be addressed directly, not by hoping that independent banks answering to shareholders are going to invent community building investment for the rewards of opening a new branch in Scarsdale. Algebra is no short cut for common sense.

**Conclusion**

This paper has set down some of the difficulties involved in the CRA. It has shown, in particular, that the effort to find an objective measure of CRA compliance relies directly, and entirely, on measuring how much money was put out for investment in the under-served community. The measures do not ask if the investment is for better roads, more reliable electric power, or another liquor store. The measures do not ask how many jobs were created. Yet if these are the true aims of the CRA, why should a bank be involved in devising them? That is not the reason banks were created. That is not what banks are good at. If government can come up with initiatives, let the government attempt them. A further benefit of governmental responsibility is that the initiaves and the results can be evaluated in the political marketplace, where it belongs.

**References**


4. Jackson, Elizabeth Rodriguez. Coping with change in the Financial Services Industry. Federal Reserve Bank of New York. In reference to the Gramm-Leach-Bliley Act (GLBA) of 1999, in specific the Community Reinvestment Act provisions and the section 711, the CRA Sunshine Requirements. I think the Fed was particularly pleased to have a woman sensitive to minority interests make this report.

The Intuition and Methodology of Value at Risk

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Abstract

Value at Risk (VaR) is the maximum dollar portfolio amount that can be lost in a given period of time with a specified level of confidence — usually 5%. VaR has become a valuable tool that financial managers can use to measure market risk. The three basic VaR methodologies are Historical, Parametric and Simulation VaRs. Each has advantages and limitations, as well as ease of application under varied circumstances. In theory, the three should generally give equal values and any differences in their computed values are attributable to modeling issues and violations of assumption. Trends in VaR should be noted and explained. More problematic than the actual number and/or differences in the number is the array of possible realizations during the “other 5% of the time.” If the other 5% of the time is well behaved, then those realizations should be anticipated and easily dealt with. On the other hand, if the other 5% of the time is not well behaved, then those realizations could be catastrophic and could lead to the demise of the enterprise.

Introduction

Risk, and how to measure it, are topics of great interest and increasing concern in the financial services industry these days. This comes as a consequence of macroeconomic turmoil and concomitant financial meltdowns in fixed income, mortgage and mortgage-backed securities over the past 12 months, as well as the spill-over into the equity markets, which has eroded personal and corporate balance sheets.

There are many sources of risk to an enterprise. Risk can be interest-rate related, exchange-rate related, political or geopolitical, weather risk, macroeconomic, model risk, accounting, fraud and malfeasance related, among others. Witness the financial crises the financial community has suffered postwar in general, but post 1970s in particular: oil price shocks (1973), Black Monday (Oct. 1987), the Mexican Peso crisis (1994), the Asian Crisis (1997), the Russian political crisis (1998), Long Term Capital Management (1998) the World Trade Center terrorist attack (2001) and others, all of which were the result of specific types of risk mentioned above. The importance of risk metrics increases with the size of the enterprises as well as increased risk associated with financial leverage and the use growth of derivative contracts.

Financial managers have seized on Value at Risk (VaR), a tool introduced in the 1980’s, to monitor and manage market risk. VaR summarizes in one number the market risk of an enterprise – it is the maximum amount that an enterprise can lose in a given period of time with a given level of confidence. The purpose of VaR is to provide an analyst, an executive, a risk manager or a regulator with:

1. a guide as to the risk of market risk of an enterprise at any particular moment, as well as
2. the trends in the market risk of an entire enterprise over time.

VaR is a mathematical and statistical methodology with a number of assumptions – some quite restrictive and some less so. The accuracy of the VaR computation depends upon the degree to which the assumptions hold. If the assumptions do not hold, then
the VaR will not be very accurate and the unknowing financial manager can be lulled into a false sense of security.

*VaR summarizes ... the market risk of an enterprise – it is the maximum amount that an enterprise can lose in a given period of time with a given level of confidence.*

This paper discusses the intuition and methodology associated with several VaR metrics and identifies limitations and assumptions which underlie their accuracy. The broadcast and print media alike have wondered aloud how, given the level of expertise of so many large and sophisticated organizations, such large portfolio losses could have occurred where risk was presumably being adequately monitored. This paper helps identify reasons that could have happened and how, as the question has come to be framed, “so many smart people could get it wrong”.

**VaR – The Basic Goal**

A discussion of Value at Risk (VaR) best begins with a definition reiterated from above:

Value at Risk (VaR) - the greatest portfolio loss that could be sustained in any given period of time for a given level of confidence.

Consider an investment portfolio consisting of stocks, bonds and other securities. Exhibit 1, below, displays a histogram of the one-day dollar returns of the current portfolio, using current portfolio weights over the last 500 days, with a normal, bell-shaped curve overlaid upon the histogram.

**Exhibit 1**
**Histogram of Portfolio Dollar Returns and One-day/5% VaR**

Notice the vertical line dropped at -$16,449.63. This line separates the bottom 5% of the portfolio dollar returns per day, from the top 95% of the days. This value is the 5%, 1-day VaR, i.e. the portfolio loss in dollars, such that there is only a 5% probability that the portfolio could lose more than that amount in one day. Even though the cutoff is technically negative, the VaR estimate is expressed as a positive number.

VaRs are typically computed for 1, 10 or 20 days using 1% or 5% levels of probability. Monthly (20 trading days) VaRs are common for portfolios where daily performance is not readily available. For example, many hedge fund investors are only provided monthly returns by their managers without transparency into the underlying portfolio securities weights and returns, all of which might otherwise facilitate daily analysis of performance. Here, the hedge fund investor would be forced to make use of a monthly VaR. On the other hand, the hedge fund manager that knows the composition of the underlying portfolio would use a 1-day VAR.

VaR methodologies produce a number, hopefully an accurate number, which executives and risk managers can be comfortable with and have a sense of the risk of the enterprise at
any specific moment in time. In all likelihood, that number is approximately accurate. The true number might be different, but within a margin of error, and probably a small difference relative to the capital of the firm. Managers can monitor the VaR number in a time series context. VaR numbers exhibiting a drift, however slight, up or down, for example the dotted lines shown in Exhibit 2, may be important in terms of the firm increasing or decreasing in risk. On the other hand, trendless variation in VaR, e.g., the solid line in Exhibit 2, can be considered statistical noise.

Exhibit 2
VaR Time Series Charts – Stationary, Trend Up, Trend Down

While the VaR number is important, and trends in VaR are equally so, what is problematic is what happens “the other 5% of the time,” that is, those periods when realized dollar loss is greater than the VaR estimate. It is not enough to know that a loss will be sustained more than $x, 5% of the time. It is more important to know, or at least worry about, the magnitude of the losses in those 5% of the instances. At the very minimum, one should be able to answer the question as to whether the tail regions, i.e. the other 5% of the time, are predictable or erratic. Often times it is quite difficult and at other times simply impossible, to know. The issue is akin to the insurance industry. While it might be known that 2% of the written hurricane policies produce a claim in a given year, it is the variation in those 2% of the claims which could be catastrophic. This issue, as well, will be elaborated upon below.

Three VaR Methodologies

There are three basic VaR methodologies, each with varying degrees of complexity:
1. Historical and Historical/Parametric VaR
2. Parametric VaR
3. Simulated Historical and Simulated Parametric VaR

Theoretically, any of the three methods could be applied to any portfolio. The reality, however, is that some methods are more easily and more appropriately applied in particular situations than others. The choice of method is largely affected by the availability and type of data. For securities with a long and liquid history, the historical approach is the preferred method. For more complicated securities, derivative instruments in particular and where the security is thinly traded, the parametric approach might be preferred because it can capture the effect of the underlying variables. For proposals of a newly designed product, especially one with asymmetric return streams, the simulation methods might be preferred.

Also in theory, the estimates using all methodologies should be the same or close. The reality here is that statistical issues and model intricacies will render different estimates. The causes of any differences should be investigated and understood.

Each method relies on a set of historical data as inputs for the computations and the interpretation of the estimate is the same regardless of the method employed. Each method is largely an exercise in statistics, economics, and portfolio theory, and makes moderate to intensive use of computer hardware and software, depending upon the
number and size of portfolios. The difference between the methods lies in what is done with the historical data to arrive at a VaR. Where possible, most risk managers compute at least two of the four VaRs for at least two time horizons. One VaR is the primary metric and the second is used as a check.

It is more important to know, or at least worry about, the magnitude of the losses in those 5% of the instances.

Two common initial steps

The following two initial steps are common for the for VaR methods:

1. Choose the time horizon, i.e. 1-day, 10-day, or 20-day. Choose the confidence limit, i.e. 95% or 99% for the VaR computation.

2. Establish the sample size/number of periods. With the historical VaR or the parametric historical VaR methods, the issue involves a tradeoff between the benefits of larger sample size vs. with the decrease in relevancy associated with stale data. Larger sample sizes in statistical analysis are better, other things equal, because they have lower margins of error. On the other hand, old data can be irrelevant and obsolete, especially at the stock security level. For example, daily observations from 10 years ago might have no relevance for a company that has tripled in size or has undergone some type of business model transformation.

The sample size used should be large enough to include at least five observations in the tail region. Therefore, if five observations constitute 1% of the observations, then the number of daily observations should be 500. This can present complications if the analysis requires, for whatever reason, monthly data.

For each of the following examples detailing the computations of VaR, we assume a portfolio of $10M with allocated 40%, 35%, and 25% to Merck (MRK), Microsoft (MSFT), and ExxonMobil (XOM), respectively.

Historical VaR

Historical VaR is the easiest of the three VaR methodologies. Beyond the common initial two steps identified above, the following steps are required:

3. For each security currently in the portfolio, obtain historical rates of return
for each time period in the time horizon and granularity of each time period. It is immaterial that the assets were not in the portfolio during the historical period.

4. Apply each historical period return to the current dollar value of each security. This results in a dataset of dates (rows) by security dollar return (columns) per period.

5. Sum across periods to find the portfolio dollar return per period. This results in a column of portfolio dollar returns for each date for the current dollar holdings of each security.

6. Sort the historical portfolio dollar returns in ascending order. Historical VaR estimates are the cutoffs established at the lowest 1% and lowest 5% values. In the case of 1000 observations, the 1% cutoff would be the 10th value in the ascending order and the 5% cutoff would be the 50th value. These are the VaR numbers reported to management. The inference to be had is that, assuming that next period’s return pattern resembles or comes from the same distribution as the historical distribution, there is only a 1% and 5% probability that the portfolio will lose more than this value.

Exhibit 3 displays columns for date, adjusted closing stock prices, and percentage security returns and dollar security returns. The table includes a final column on the extreme right as the sum of the dollar returns for the portfolio for that month based upon the 40%, 35%, and 25% holdings of MRK, MSFT and XOM, respectively.

<table>
<thead>
<tr>
<th>Date</th>
<th>Adjusted Closing Prices</th>
<th>Percentage Returns</th>
<th>Dollar Returns</th>
</tr>
</thead>
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<td></td>
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<td>msft</td>
<td>xom</td>
</tr>
<tr>
<td>1/3/2006</td>
<td>30.53</td>
<td>26.10</td>
<td>56.18</td>
</tr>
<tr>
<td>1/4/2006</td>
<td>30.88</td>
<td>26.23</td>
<td>56.27</td>
</tr>
<tr>
<td>1/5/2006</td>
<td>30.93</td>
<td>26.25</td>
<td>55.99</td>
</tr>
<tr>
<td>1/6/2006</td>
<td>31.38</td>
<td>26.12</td>
<td>57.07</td>
</tr>
<tr>
<td>1/7/2006</td>
<td>30.88</td>
<td>26.23</td>
<td>56.27</td>
</tr>
<tr>
<td>1/8/2006</td>
<td>31.19</td>
<td>26.54</td>
<td>57.91</td>
</tr>
<tr>
<td>1/9/2006</td>
<td>31.21</td>
<td>26.39</td>
<td>57.30</td>
</tr>
<tr>
<td>1/10/2006</td>
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<td>1/11/2006</td>
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<td>59.13</td>
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<td></td>
<td></td>
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<td>12/21/2007</td>
<td>59.15</td>
<td>36.06</td>
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<td>12/22/2007</td>
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<td>12/26/2007</td>
<td>58.11</td>
<td>35.60</td>
<td>93.29</td>
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</table>

Exhibit 3 displays columns for date, adjusted closing stock prices, and percentage security returns and dollar security returns. The table includes a final column on the extreme right as the sum of the dollar returns for the portfolio for that month based upon the 40%, 35%, and 25% holdings of MRK, MSFT and XOM, respectively.
Exhibit 4 displays the dollar return data sorted in ascending order of the portfolio dollar return. The two numbers that are boldfaced — $149,387 and $292,193 — are the cutoffs for the 5% and 1%, respectively. These are the historical value at risk cutoffs.

Note two important items regarding historical VaR. First, no assumptions are made about the mean, dispersion, skewness, kurtosis or correlation of the underlying securities or the aggregated portfolio. The skewness, kurtosis and correlations between securities are embedded in the historical series. This is an advantage of historical VaR over the parametric and simulation VaR. Second, to reiterate the familiar theme and extraordinarily important point, the computed VaR equal to -$149,387 is certainly of interest. Hence 5% of days should have a portfolio loss less than this number. What should be of even greater interest is what can happen for those 5% of the periods where the loss is greater than minus $149,387. Inspection of those values is, by construction, possible with this method.

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<th>Date</th>
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<tr>
<td>3/12/2007</td>
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<td>12/26/2007</td>
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<td>61,185.47</td>
<td>29,925.85</td>
<td>148,331.11</td>
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</tr>
</tbody>
</table>
Historical/Parametric VaR

This hybrid method uses the mean and standard deviation of the historical distribution, and assuming a normal distribution, i.e. skewness and kurtosis equal to 0, backs out the cutoff of the bottom 5% of the distribution from the top 95% or bottom 1% of the distribution from the top 99% of the distribution. Exhibit 5 displays descriptive statistics for the historical data.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10,731</td>
<td>$98,553</td>
<td>-.15</td>
<td>2.13</td>
</tr>
</tbody>
</table>

As the mean and standard deviation are $10,731 and $98,553, respectively, values of X can be solved for, as follows:

$$-1.65 = \frac{X - 10,731}{98,553}$$

$$X = - $151,881$$

$$-2.33 = \frac{X - 10,731}{98,553}$$

$$X = - $218,897$$

Therefore there is a 5% probability of losing more than $151,881, and a 1% chance of losing more than $218,897. The crucial assumption in this method is that the dollar return data is normally distributed. That the values from the historical VaR and the hybrid are not the same implies that the data contains some degree of skewness or kurtosis. Indeed the skewness and kurtosis of the historical data were -.15 and 2.13, respectively. The kurtosis parameter of 2.13 implies serious kurtosis and explains why the 1% Historical VaR limit is so much greater than the 1% VaR limit for the parametric historical method.

Parametric VaR

The basic Parametric VaR method assumes that the return stream is normally distributed with constant mean and standard deviation, and with skewness and kurtosis equal to 0. As was the case with Historical VaR, the analysis centers on the dollar return formulation. Following the two initial common steps listed on page 42, the following are required:

3. Find the mean and standard deviation return for each security in the portfolio.
4. Compute a correlation matrix for the security returns.
5. Aggregate the individual returns to a historical portfolio return and standard deviation using the Markowitz expansion.
6. Use the normal distribution formula to “back out” a VaR estimate.
For example, consider the same $10 million portfolio of three assets. Return means, standard deviations and correlations are displayed in Exhibit 6 and 7, below.

**Exhibit 6**

Descriptive Statistics of Returns

<table>
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<th></th>
<th>mrk</th>
<th>msft</th>
<th>xom</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0014</td>
<td>0.0007</td>
<td>0.0011</td>
</tr>
<tr>
<td>std dev</td>
<td>0.0129</td>
<td>0.0137</td>
<td>0.0135</td>
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</tbody>
</table>

**Exhibit 7**

Correlation Matrix of Returns

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<th>msft</th>
<th>xom</th>
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<tr>
<td>msft</td>
<td>0.3068</td>
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<tr>
<td>xom</td>
<td>0.2730</td>
<td>0.3521</td>
<td>1</td>
</tr>
</tbody>
</table>

Substituting parameters into Equations 1 and 2, the estimates for the portfolio mean and standard deviation are:

Equation 1

\[ r_p = w_a r_a + w_b r_b + w_c r_c \]

\[ r_p = .4 \times 0.0014 + .35 \times 0.0007 + .25 \times 0.0011 \]

\[ r_p = .0010731 \]

Equation 2

\[ \sigma_p^2 = w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + w_c^2 \sigma_c^2 + 2w_a w_b \sigma_a \sigma_b \rho_{ab} + 2w_a w_c \sigma_a \sigma_c \rho_{ac} + 2w_b w_c \sigma_b \sigma_c \rho_{bc} \]

\[ \sigma_p^2 = .4^2 \times 0.0129^2 + .35^2 \times 0.0137^2 + .25^2 \times 0.0135^2 + 2 \times 0.4 \times 0.35 \times 0.0129 \times 0.0137 \times 0.3068 + 2 \times 0.4 \times 0.25 \times 0.0129 \times 0.0135 \times 0.2730 + 2 \times 0.35 \times 0.25 \times 0.1037 \times 0.0135 \times 0.3521 \]

\[ \sigma_p = .009854 \]

Applying each result to $10,000,000, the daily portfolio mean and standard deviation are $10,731 and $98,536, respectively. Substituting to the normal distribution, as follows,

\[ -1.65 = \frac{X - 10,731}{98,536} \]

\[ -2.33 = \frac{X - 10,731}{98,536} \]

\[ X = -$151,853 \]

\[ X = -$218,857 \]

Therefore, the 5% 1-day VaR is -$151,853 and the 1% 1-day VaR is -$218,857.

This has the interpretation that there is a 5% chance that the portfolio could lose more than $151,853 in a single day and a 1% chance that the portfolio could lose more than $218,857.

As indicated above, these computations assume the distribution of dollar returns to be normally distributed. If the data is skewed to a serious degree, then the computed VaR should be questioned. Unlike the historical VaR, the parametric VaR does not afford one the opportunity to inspect those actual losses below the VaR limit, a disadvantage of this method.
Monte Carlo Simulation

This method is most complicated.

• A random number is drawn from a specified distribution (normal or otherwise) and a rate of return associated with that number is determined from the mean and standard deviation for a security for a particular day.

• A second random number is then drawn. A (linear) transformation associated with the correlation between the first security and the second is applied to establish the second rate of return.

• A third random number is a transformation of the correlation applied to the first, and the associated return determined.

This is done for each security at each trial. The trials are recorded in time series for 1000 trials. As with historical VaR, the simulated returns are applied to the dollar holdings for the day. VaR estimates are then established either by:

1. sorting the simulated portfolio returns in ascending order, and determining the cutoff of the bottom 1% or 5% of the simulated trials, or

2. computing the mean and standard deviation of the simulated portfolio returns, and then using the normal distribution as establishing the bottom 5% or 1% cutoff.

A series of 500 random numbers are applied to the mean and standard of Merck deviation to simulate a return for the day. A linear relationship is established between rates of return for Merck and ExxonMobil and between Merck and Microsoft. Having established the simulated return for Merck, the simulated returns for ExxonMobil and Microsoft can be determined by applying the linear (beta) relationship adjusted by the addition of a random component established by the standard error.

The first 26 sorted Monte Carlo Simulation VaR computations appears in Exhibit 8, the 5% and 1% VaR limits of -$165,183.34 and -$249,297.53, respectively, shown in boldface.

Exhibit 8
Monte Carlo VaR Simulation Results

<table>
<thead>
<tr>
<th>trial no.</th>
<th>Mrk</th>
<th>msft</th>
<th>xom</th>
<th>portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>-81937.32</td>
<td>-124098.51</td>
<td>-88641.79</td>
<td>-294677.63</td>
</tr>
<tr>
<td>336</td>
<td>-47499.70</td>
<td>-143317.74</td>
<td>-102369.81</td>
<td>-293187.25</td>
</tr>
<tr>
<td>200</td>
<td>-105884.47</td>
<td>-104407.45</td>
<td>-74576.75</td>
<td>-284868.67</td>
</tr>
<tr>
<td>63</td>
<td>-126527.73</td>
<td>-74513.42</td>
<td>-53223.87</td>
<td>-254265.03</td>
</tr>
<tr>
<td>15</td>
<td>-59894.42</td>
<td>-110485.15</td>
<td>-78917.96</td>
<td>-249297.53</td>
</tr>
<tr>
<td>108</td>
<td>-130866.21</td>
<td>-65573.34</td>
<td>-57201.51</td>
<td>-231677.64</td>
</tr>
<tr>
<td>103</td>
<td>-74012.31</td>
<td>-96258.94</td>
<td>-68756.39</td>
<td>-239027.63</td>
</tr>
<tr>
<td>329</td>
<td>-133695.17</td>
<td>-58950.27</td>
<td>-42107.33</td>
<td>-234752.77</td>
</tr>
<tr>
<td>484</td>
<td>-94384.02</td>
<td>-80082.11</td>
<td>-57201.51</td>
<td>-231677.64</td>
</tr>
<tr>
<td>216</td>
<td>-120609.27</td>
<td>-64539.17</td>
<td>-46099.41</td>
<td>-231247.85</td>
</tr>
<tr>
<td>39</td>
<td>-35428.13</td>
<td>-103627.22</td>
<td>-46999.41</td>
<td>-213247.85</td>
</tr>
<tr>
<td>108</td>
<td>-130866.21</td>
<td>-65573.34</td>
<td>-57201.51</td>
<td>-231677.64</td>
</tr>
<tr>
<td>178</td>
<td>-85798.58</td>
<td>-65477.74</td>
<td>-46769.82</td>
<td>-198046.15</td>
</tr>
<tr>
<td>210</td>
<td>-20996.71</td>
<td>-102016.76</td>
<td>-72869.11</td>
<td>-195882.58</td>
</tr>
<tr>
<td>93</td>
<td>-13739.29</td>
<td>-103499.04</td>
<td>-73927.88</td>
<td>-191166.21</td>
</tr>
<tr>
<td>300</td>
<td>-34667.22</td>
<td>-89010.84</td>
<td>-63579.17</td>
<td>-187257.22</td>
</tr>
<tr>
<td>270</td>
<td>-78986.35</td>
<td>-62262.95</td>
<td>-44473.54</td>
<td>-185722.84</td>
</tr>
<tr>
<td>148</td>
<td>-42113.92</td>
<td>-82885.45</td>
<td>-59203.89</td>
<td>-184203.26</td>
</tr>
<tr>
<td>407</td>
<td>-124270.89</td>
<td>-32956.90</td>
<td>-23540.64</td>
<td>-180768.42</td>
</tr>
<tr>
<td>195</td>
<td>-1038.95</td>
<td>-99697.52</td>
<td>-71212.52</td>
<td>-171948.99</td>
</tr>
<tr>
<td>262</td>
<td>-57633.85</td>
<td>-65450.12</td>
<td>-46750.09</td>
<td>-169834.06</td>
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<tr>
<td>296</td>
<td>-166556.53</td>
<td>-742.48</td>
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<td>456</td>
<td>-48042.94</td>
<td>-68777.46</td>
<td>-49126.75</td>
<td>-165947.16</td>
</tr>
<tr>
<td>251</td>
<td>-73174.13</td>
<td>-53672.04</td>
<td>-38337.17</td>
<td>-165183.34</td>
</tr>
<tr>
<td>350</td>
<td>-63022.31</td>
<td>-57008.22</td>
<td>-40720.15</td>
<td>-160750.68</td>
</tr>
</tbody>
</table>
On the other hand, Monte Carlo Parametric Simulation can be computed based upon the mean and standard deviation of the series computed above. The mean and standard deviation, respectively, are $10,795 and $107,849. The 5% and 1% VaR estimates can be computed as follows:

\[-1.65 = \frac{X - 10,795}{107,849} \quad -2.33 = \frac{X - 10,795}{107,849}\]

Solving, the 5% and 1% VaR estimates are $167,157 and $240,495, respectively.

...if the quantitative professional’s model does not accurately replicate reality, then unanticipated realizations will occur on the positive and negative side...

Comparison of the Basic Models

It was mentioned above that under certain circumstances all four of the VaR methodologies will produce the same number. Those circumstances are that the distribution of the portfolio dollar or percentage returns are either perfectly or approximately normally distributed; that is, the skewness and kurtosis of the portfolio dollar returns are 0 and there are no outliers. VaR estimates differ because one of these two are not true, and when such a difference occurs, the cause of the differential must be understood. Once the cause is determined, the basic VaR model can be modified to correct to the discrepancy.

Exhibit 9 compares the 5% and 1% VaRs for both the historical and parametric methods. Except for the Simulation VaRs, the values are roughly the same.

Exhibit 9
Comparison of Historical, Parametric, and Simulation VaR Estimates

<table>
<thead>
<tr>
<th>Method</th>
<th>5% VaR</th>
<th>1% VaR</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical</td>
<td>$149,387</td>
<td>$292,193</td>
<td>-.215</td>
<td>2.13</td>
</tr>
<tr>
<td>Historical/Parametric</td>
<td>$151,881</td>
<td>$218,897</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parametric</td>
<td>$151,853</td>
<td>$218,857</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>$160,228</td>
<td>$230,973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation/Parametric</td>
<td>$167,157</td>
<td>$240,495</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that skewness associated with the Historical VaR, that is -.215, is inconsequentially different from 0, but that the kurtosis value is quite significant. Hence, the 5% VaR is quite close to the other computations, but the 1% VaR limit is quite far off, owing to the extreme events that occurred during the historical period. The Historical/Parametric and Parametric VaRs are virtually identical. These results are to be expected, since both assume a normal distribution.

That the VaR estimates in Exhibit 9, except for the Simulation VaR, are for the most part consistent owes to the relatively uncomplicated nature of the securities and their return behavior. As the securities become more complicated – bonds, futures, options on equities, options on bonds, options on bond futures, to cite a list of increasing complexity — the modeling complexity increases, and the degree to which the estimates diverge will increase.

What is quite problematic and a source of anxiety to the financial manager is what could happen the other x% of the time. Consider the Historical 1% VaR of $292,123. Assuming that metric to be accurate, then by definition 1% of the daily realizations should experience a loss greater than $292,123. What is unknown is the extent of the losses under that scenario. The question of huge financial consequence
is whether the losses on those 1% of the days are concentrated around $292,123, marginally greater than $292,123, or seriously greater than that.

**Stress Testing and Specific Day Measurements**

VaR is an attempt by management to model the risk of an enterprise and understand what could happen under worst case market scenarios. VaR limits of 5% or 1% levels of confidence are conventional benchmarks chosen to identify the maximum portfolio loss that might occur with a given level of confidence. Therefore, by definition, it is relatively common to lose more than the 5% VaR (1 out of 20 days) and less-so for 1% (1 out of every 100 days).

In essence VaR is an attempt to model human behavior as instigated by real economic, geopolitical, environmental phenomena affecting markets. But at the end of the day, and in spite of the intricate modeling work associated with the three basic models delineated above, VaR amounts to sophisticated guesswork. VaR is a best guess subject to attendant assumptions and complications as represented by skewness, leptokurtosis and outliers (especially), and conditional volatility and conditional correlation.

Recognizing the limitations of the accuracy of the 5% and 1% VaRs, and worse yet, by the attendant difficulty of modeling what could happen those other 5% of the time, most vendor VaR software packages have been engineered to stress test the portfolio, i.e. compute pro forma portfolio returns for significant changes in interest rates, exchange rates, oil prices or other macroeconomic events which could negatively affect the portfolio value. The vendor packages will “stress test,” that is, move these input variables to an extreme of six standard deviations, to assess the effect on the portfolio. In essence this is an extended version of Parametric VaR.

Another tool in vendor software packages which facilitate managers’ understanding of what could happen the “other 5% of the time” is the computation of proforma VaRs for specific days in the history of interest – days with geopolitical episodes inducing large security or market moves of four standard deviations or more. These are days such as October 19, 1987, September 11, 2001, the Kobe Earthquake in Japan, or the Russian Revolution.

**Summary and Conclusions**

Value at Risk is a valuable tool used by financial services managers to monitor and manage market risk. VaR, an absolute dollar value, was defined as an estimate of the maximum loss that could take place over a specified period of time with a given level of confidence, usually 5% or 1%.

This paper focused on three VaR methodologies and issues regarding their construction. Properly constructed, the VaR values should be approximately the same. Any differences will be due to a violation from an assumption in the historical, parametric or simulation computational methods. Analysts should understand the causes of the differences to some comfort level.

Two issues were specifically raised. The first is the accuracy of the 5% or 1% cutoff. Aside from the minor differences produced by the three VaR methods, the number will be, for all practical purposes, approximately correct and should indeed be an adequate estimate of the cutoff. Whether that number is truly slightly higher or lower is probably not of great consequence. Any trend or persistent drift in that number, however, should be investigated.

By definition, one of twenty days — once a month — the portfolio return will be less than the 5% computation. VaR amounts to sophisticated guessing, but information allowed by sophisticated guessing is much
better than the days of no information at all. Therefore, the manager is better off with that information.

The second issue, however, i.e. what happens the other 5% of the time, has greater financial consequence and should be of greater concern. If the distribution of returns is normally distributed, then the possibilities in the “other 5% of the time” can be estimated fairly accurately from the parametric distribution or historical/parametric distribution. If the returns are not normally distributed, then the possible realizations cannot be estimated accurately, and anything can happen during the “other 5% of the time” and probably will, sooner or later. These realizations are best hinted at using historical VaR or one of the simulation methods.

Mathematical models, in this case VaR models, are abstractions from reality and are used to project or simulate portfolio outcomes. Try as they might, if the quantitative professional’s model does not accurately replicate reality, then unanticipated realizations will occur on the positive and negative side, with negative outcomes much more likely to hit the broadcast and print media headlines than positive outcomes. It is when the models do not replicate reality that financial managers get a false sense of security as to the risk of the enterprise with the possibility of adverse portfolio outcomes. A severely negative portfolio outcome which was a consequence of a model which did not adequately replicate reality, perhaps due to skewness or leptokurtosis, is one explanation for “how so many people could get it wrong.”

References

Abstract

Many nations, large and small, use an Input-Output Model to study the interdependence of the production plans and activities of the many industries which constitute an economy. But few, if any, have used this methodology for forecasting purposes.

This paper outlines a procedure for extending the static (open-loop and time independent) model to a dynamic and recursive (closed loop and time dependent) model suitable for forecasting. The procedure begins with the definition and analysis of the open-loop static model (i.e. the existing input-output model) which is then modified and converted to a closed-loop (recursive) model which is suitable for forecasting.

This conversion is done gradually, introducing only one possible change at a time, thus generating eight (8) total possible models. The cost associated with each of these eight models is also derived and it is found that the cost of the full recursive model is at least twice the cost of the simple static model.

Can such a cost be justified? The answer to this question depends on the ability of the revised input-output methodology to provide useful and credible forecasts.

Introduction

Input-Output analysis seeks to take into account the interdependence of the production plans and activities of the many industries which constitute an economy [4]. This interdependence arises out of the fact that each industry employs the outputs of other industries as its raw materials. Similarly, its output is often used by other producers as a productive factor, sometimes by those very industries from which it obtained its ingredients.

Suppose:

\[ X_i = \text{quantity of good } i \text{ produced by industry } i \text{ in a given period} \]
\[ Y_{ij} = \text{number of units of good } i \text{ required by industry } j \text{ in the given period} \]
\[ b_i = \text{exogenous demand for good } i \text{ for a given period} \]

Therefore, in order for industry \( i \) to meet the demands on it, it must produce at least \( X_i \) units, where:

\[ X_i = Y_{i1} + Y_{i2} \ldots + Y_{in} + b_i \] (1)

For industry \( j \) to produce \( X_j \) units it would require inputs from other industries \((i, j, k, \ldots )\). The number of units of goods that it requires from these industries depends, to a large extent, on the technology of these industries [2].

The basic assumption of Input-Output Analysis, for which we are indebted to Professor Leontief, states that: “... in any productive process all inputs are employed in rigidly fixed proportions and the use of these inputs expands in proportion with the level of output.” Another way of stating this is to say that “the amount of good \( i \) required to produce good \( j \) is directly proportional to the
amount of good j produced.” These statements imply that

\[ Y_{ij} = a_{ij} X_j \]  \hspace{1cm} (2)

where:

\[ a_{ij} = \text{constant of proportionality, depending on the technology of industry } j. \]

Basically, the Input-Output Analysis consists of nothing more complicated than the solution of a set of n simultaneous linear equations in n variables. These equations come about when the basic assumption, as expressed by equation (2), is substituted into equation (1). The result is [1, 8, 13]:

\[
\begin{bmatrix}
(1 - a_{11}) & -a_{12} & -a_{13} & \ldots & -a_{1n} \\
-a_{21} & (1 - a_{22}) & -a_{23} & \ldots & -a_{2n} \\
-a_{31} & -a_{32} & (1 - a_{33}) & \ldots & -a_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_{n1} & -a_{n2} & -a_{n3} & \ldots & (1 - a_{nn})
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
\vdots \\
X_n
\end{bmatrix}
= 
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
\vdots \\
b_n
\end{bmatrix}
\]  \hspace{1cm} (3)

or: \((I - A)X = B\)  \hspace{1cm} (4)

\[ X = (I - A)^{-1}B \]  \hspace{1cm} (5)

The model described above is static since everything is rigidly fixed, if the \(a_{ij}\) coefficients are constant. It is a useful tool in understanding the interactions between the many industries of an economy. However, except for small time periods, it is quite likely that the coefficients of the model will change, even though the extent and rate of this change may not be known because not enough pertinent data may exist for measuring this change. As is, the model allows only a single look at the interactions among the segments of an economy. To obtain more looks we need more data, at different times.

If new sets of \(a_{ij}\) coefficients and exogenous demands \(b_i\) can be obtained, the model can be solved several times to obtain updated solutions. But the model remains open since no feedback is possible under the assumptions of the static model. However, the need for more data, implies increased cost.

To convert the open-loop model into a closed-loop model, suitable for forecasting, the output at time \((t + 1)\) must be related to the output at time \(t\). This can be accomplished by introducing the concept of “Stocks of Capital Goods” which, essentially, allows the buildup of Inventory, something that represents a departure from the underlying assumptions of the static model [12].

The static model can also be extended in other directions, thus converting the “simple static” model into a “complete recursive” model, but with a correspondingly higher cost.

Before we proceed with the development of the extended models it is necessary to pause briefly and derive the cost function associated with the static model. This cost function will serve as the reference against which the cost of the extended models will be compared to.

1. Cost Associated with the Static Model

The solution of equation (5) is procedural [9, 10, 11] once the \(a_{ij}\) and \(b_i\) coefficients have been evaluated, even if the size of the matrix which needs to be inverted is very large. The major cost of the static model is due to the difficulty associated with the determination of the technological coefficients \(a_{ij}\) and the exogenous demands \(b_i\) [3].

To obtain an estimate of the cost of the static model, let us define the costs \(C_{ij}\) and \(C_i\) where:

\[ C_{ij} = \text{Cost of obtaining the technological coefficient } a_{ij} \]

and

\[ C_i = \text{Cost of obtaining the exogenous demand } b_i \text{ for industry } i \]
Then, the cost of obtaining all the technological coefficients can be defined by:

$$C_{\text{tech coef}} = \begin{bmatrix} C_{11} & C_{12} & \ldots & C_{1n} \\ C_{21} & C_{22} & \ldots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \ldots & C_{nn} \end{bmatrix} = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}$$  \hspace{1cm} (6)

Similarly the cost of obtaining all the exogenous demands can be defined by:

$$C_{\text{exog. dem}} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix} = \sum_{i=1}^{n} C_i$$  \hspace{1cm} (7)

The total cost of the static model is equal to the sum of the costs of the technological and exogenous demand coefficients and it, therefore, is given by:

$$C_{\text{total cost}} = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} + \sum_{i=1}^{n} C_i$$  \hspace{1cm} (8)

If we now make the assumption that $C_{ij} = C_a$ for all $i$ and $j$, and $C_i = C_b$ for all $i$, then equation (8) can be rewritten as:

$$C_{\text{total cost}} = n^2 C_a + n C_b = n(n+1)C_b$$  \hspace{1cm} (9)

$$= n(n+1)C_b$$  \hspace{1cm} (10)

The last equation, which represents the cost of the static model under the stated assumptions, will be used as the reference against which the cost effectiveness of the extended models will be compared to.

2. Extensions to the Static Input-Output Models

To render the static model more useful for prediction, it can be modified by relaxing the “fixity” conditions of the technological coefficients. Also the model can be formulated in a “closed-loop” form so that future predictions depend on past predictions, thus resulting in a recursive model [15]. In this paper the extensions to the static model proceed in three directions:

- The technological coefficients $a_{ij}$ are made function of time (T)
- The technological coefficients $a_{ij}$ are made random variables (R)
- The concept of “stock of capital goods” is introduced, which converts the “open-loop” static model into a “closed-loop” and recursive one suitable for forecasting (S).

The extensions can be made in any of these three directions, or combinations of them. Since there are three possible areas of extension with two levels each (i.e., present in the model; not present in the model), there are $2^3 = 8$ possible models. Using the letter symbols for each of the three model extension directions, the eight possible models are [5, 6, 7]:

1) __ __ __ present static model
2) __ __ T  $a_{ij} = f(\text{time})$
3) __ R __ $a_{ij}$ is a random variable
4) __ R __ T  $a_{ij} = f(\text{time})$ & $a_{ij}$ is a random variable
5) S __ __ “simple” recursive model; “stocks of Capital Goods”
6) S __ T  “simple” recursive model with $a_{ij} = f(\text{time})$
7) S R __ “simple” recursive model with $a_{ij}$ a random variable
8) S R T  “complete” recursive model

The basic assumption of Input-Output Analysis states:
“...in any productive process all inputs are employed in rigidly fixed proportions and the use of these inputs expands in proportion with the level of output.”

Professor Wassily Leontief
3. Other Open-Loop Models and their Costs

Model 1 is the present static model which has been discussed in some detail above. Models 2, 3 and 4 represent various assumptions on the $a_{ij}$ coefficients which are shown on Exhibits 1 and 2.

Since we live in an era of great technological change in which manufacturing and industrial processes change continuously, it is apparent that the assumption of fixed $a_{ij}$ is not very realistic, especially when the period of prediction is relatively long. The $a_{ij}$ can be made functions of time and/or converted to random variables. This implies that functional relationships have to be obtained for each $a_{ij}$. Changes in the coefficients over time are mainly due to technological change and changes in consumer tastes which exert an effect on product mixes. The time variable should capture both these and other changes that may be introduced. Suppose that for each $a_{ij}$ we take $p$ measurements over a period of time. Then Least Square estimation can be used to find $a_{ij}(t) = a + bt$, as a function of time. To find the cost associated with this model we note that we need to evaluate matrix $A$, $p$ times. Therefore, the total cost of this model is:

\[
\text{TOTAL COST} = (n^2C_a)p + nC_b \\
= n(npC_a + C_b) \\
= n(np + 1)C \text{ if } C_a = C_b = C
\]

Based on the fact that the “fixed proportionalities” are usually arrived at as the mean of several observations over a short period of time, it seems reasonable to regard the $a_{ij}$ as random variables instead of fixed constants. If $m$ observations are needed to establish the distribution of each $a_{ij}$ at a cost of $mc_a$ if $C_i = C_j$ for all $i, j$, then the cost of matrix $A$, consisting of $n^2$ elements, is $n^2mc_a$. Therefore, the total cost of this model is:
TOTAL COST = \( n^2mC_a + nC_b \) 
= \( n(nmC_a + C_b) \)
= \( n(nm + 1)C \) if \( C_a = C_b = C \) \((13)\)

The simultaneous presence of technological change due to time and the variation due to observation errors give rise to model 4. The cost of this model is obtained as follows:

The cost of matrix A, when each \( a_{ij} \) is a random variable is \( n^2mC_a \). Since we need \( p \) observations to convert each \( a_{ij} \) to a function of time, the cost of converting each \( a_{ij} \) to both a random variable and a function of time is \( (n^2mC_a)p \).

Therefore, the total cost of the model is:

TOTAL COST = \( n^2mC_a p + nC_b \) 
= \( n(nmC_a p + C_b) \)
= \( n(nmp + 1)C \) if \( C_a = C_b = C \) \((14)\)

4. The “Simple” Recursive Model

The current output of an industry may be used for any or all of the following three purposes:
- For current consumption
- As an input in the production of some other output
- As an addition to the economy’s “stock of capital.”

The first two uses have already been included in the static model. It is the last possibility, capital investment, which is the novel feature that characterizes this model since the accumulation of outputs which are not used up when they are turned out can be essential for future production.

For the development of this model we proceed as follows:
Let
\( S_{ij}(t) = \text{Capital Stock (Inventory) of Good i held by industry j at the beginning of period t} \)
and make the assumption that
\( S_{ij}(t) = d_{ij}X_j(t) \) for all \( i, j, t \) \((d_{ij} > 0)\) \((15)\)

Then the capital stock of Good i, held by the entire economy, \( S_i(t) \), is given by:

\[ S_i(t) = \sum_{j=1}^{n} S_{ij}(t) = \sum_{j=1}^{n} d_{ij}X_j(t) \] \((16)\)

Therefore the change in capital stock of product i, from the beginning of time period t to the end of the period, is given by:

\[ \Delta S_i(t) = S_i(t+1) - S_i(t) \] \((17)\)

where \( S_i(t) \) represents “Addition to Inventory” which must be produced by industry i in time period t. The assumption that \( \Delta S_i(t) \geq 0 \), for \( i = 1, 2, ..., n \), assures that there is no depletion of productive capacity [6].

If we let \( b_i(t) \) represent the exogenous demand for product i, in time period t and assume that “production for each industry, in each time period, exactly meets product demand”, we obtain [14, 17]:

\[ X_i(t) = \sum_{j=1}^{n} a_{ij}X_j(t) + \Delta S_i(t) + b_i(t) \] \((18)\)

where:

\[ \Delta S_i(t) = \sum_{j=1}^{n} d_{ij}[X_j(t+1) - X_j(t)] \] \((19)\)

because of \((18)\) and \((19)\).

If we now define:

\[ A = [a_{ij}] \]
\[ D = [d_{ij}] \]
\[ b_i = [b_1(t), b_2(t), ..., b_n(t)]^t \]
\[ X_i = [X_1(t), X_2(t), ..., X_n(t)]^t \]
\[ S_i = [S_1(t), S_2(t), ..., S_n(t)]^t \] \((20)\)

then the model described by equation \((20)\) and \((21)\) can be written in matrix form as [16]:

\[ (I - A)X_t - DX_{t-1} + DX_t = b_t \]
\[ S_t = DX_t \] \((22)\)

Assuming that matrix D is non-singular (i.e. \( D^{-1} \) exists), the model equation can be written as:

\[ X_{t+1} = [I + D^{-1}(I - A)]X_t - D^{-1}b_t \] \((23)\)

\[ S_t = DX_t \] \((24)\)

\[ X_{t+1} = [I + D^{-1}(I - A)]X_t - D^{-1}b_t \] \((25)\)
...the cost of the full recursive model is at least twice the cost of the simple static model. Can such a cost be justified? [That] depends on the ability of the revised input-output methodology to provide useful and credible forecasts.

Therefore, if \( b_t \), and initial production vector \( X_0 \), and an initial stock \( S_0 = DX_0 \), are given the behavior of the entire economy can be determined for all future times (see Exhibit 3)

Exhibit 3
Deterministic Dynamic Model

\[
\begin{align*}
A &= \text{Matrix Containing the Technological Coefficients } a_{ij} \quad (A_{nxn}) \\
I &= \text{n x n Identity Matrix} \\
D &= \text{n x n Matrix Containing the Constants of Proportionality } d_{ij} \text{ of the “Capital Stock” Model } S_{ij}(t) = d_{ij} X_j(t) \\
B &= \text{Backward Operator}
\end{align*}
\]

The model given by equation (25) takes explicit account of what must be put aside today in order to be able to achieve our plans for tomorrow.

It is a closed-loop model (the production at time \( t + 1 \) is related to the production at time \( t \)) and as such it is suitable for forecasting.

The coefficients \( a_{ij} \) and \( d_{ij} \) are assumed to be fixed constants. When these coefficients become random variables and functions of time (complete recursive model), giving \( b_t, X_0, \) and \( S_0 \) will not be sufficient (to determine the behavior of the entire economy for all future times) because new \( A \) and \( D \) matrices need to be calculated at each time \( t \).

To find the cost associated with the “simple” recursive model we proceed as follows:

\[
\text{TOTAL MODEL} = \text{Cost of } + \text{Cost of } + \text{Cost of} \\
\text{Determining Determining Determining} \\
\text{MATRIX A} + \text{MATRIX D} + \text{VECTOR } b_t
\]

\[
\begin{align*}
&= \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} C_{dij} + \sum_{i=1}^{n} \sum_{t=0}^{n} C_{b(t)} \\
&= n^2 C_a + n^2 C_d + n(k + 1) C_b
\end{align*}
\]

If we again make the assumptions that:

\[
\begin{align*}
C_{ij} &= C_a \text{ for all } i \text{ and } j \\
C_{dij} &= C_d \text{ for all } i \text{ and } j \\
C_{b(t)} &= C_b \text{ for all } i \text{ and } j,
\end{align*}
\]

then the total cost can be written as:

\[
\text{TOTAL COST} = n^2 C_a + n^2 C_d + n(k + 1) C_b
\]

where \( k \) = number of periods for which \( b_t \) is defined.

A good measure to employ here would be Relative Cost, where:

\[
\text{RELATIVE COST} = \frac{\text{COST OF SIMPLE RECURSIVE MODEL}}{\text{COST OF STATIC MODEL}}
\]

\[
= \frac{nC(2n + k + 1)}{nC(n + 1)} = 2 + \frac{k - 1}{n + 1}
\]
The conclusion drawn from equation (29) is that the cost of the “simple” recursive model is at least twice the cost of the static model and could be more depending on the relative magnitudes of \( k \) and \( n \). Granted the cost is higher but by being willing to accept a higher cost we converted the static model to a recursive one suitable for forecasting. Whether or not one would be willing to accept this trade-off would depend on the forecasting capability of the model but this is another issue which is not the topic of this paper. Our concern now is to define the other extension models, find their cost functions, compare them to the cost of the static model and then choose the most cost effective models for further evaluation to determine their forecasting capabilities.

5. Other Closed-Loop Models and their Costs

Now that we have discussed the “simple” recursive model in some detail (A and D are non-varying matrices in this model) and have demonstrated the ability of the model to be used for forecasting purposes (at least as a concept), it will be relatively easy to introduce the time-varying and random effect concepts, which were discussed earlier, to both matrices A and D and thus move, gradually, from the “simple” to the “complete” recursive model which will include all of the changes to the present static model suggested in this paper.

We will develop the other closed-loop models by adding one change at a time to the “simple” recursive model. We begin by changing \( a_{ij} \) and \( d_{ij} \) in the A and D matrices from non-varying constants to linear functions of time (i.e. \( a_{ij}(t) = a_{ij}(0) + \delta_{ij}(t) \), and similarly for \( d_{ij}(t) \)). Schematically this model is represented also by the closed-loop diagram of Exhibit 3 [15].

The cost of this model can be obtained from the cost of the “simple” recursive model by modifying the costs of determining matrices A and D to include the effect of the measurements needed to establish the linear relationships for \( a_{ij} \) and \( d_{ij} \). Since, under the cost assumptions we made earlier, it costs \( n^2C_a \) and \( n^2C_d \) to evaluate matrices A and D respectively, when they have non-varying constants of proportionality as their elements and we required to evaluate \( p \) such matrices to establish the time relationships for \( a_{ij}(t) \) and \( d_{ij}(t) \), the total cost of the model is given by:

**TOTAL MODEL COST**

\[
(n^2C_a)\cdot p + n(k + 1)C_b = n^2p(C_a + C_d) + n(k + 1)C_b = n(2np + k + 1)C 
\]

Converting \( a_{ij} \) and \( d_{ij} \) from non-varying constants to random variables gives rise to another closed-loop model whose characteristics and cost can be obtained from the “simple” recursive model. Exhibit 3 again represents this model and the cost is obtained from the cost of the “simple” recursive model by modifying the costs of determining matrices A and D to include the effect of the observations needed to establish the distributions of \( a_{ij} \) and \( d_{ij} \). Using a reasoning similar to the one used above, the result is:

**TOTAL MODEL COST**

\[
(n^2C_a)\cdot m + n(k + 1)C_b = n(2nm + k + 1)C 
\]

The “complete” recursive model can now be obtained from the “simple” recursive model by assuming that \( a_{ij} \) and \( d_{ij} \) are both functions of time and random variables instead of being fixed constants of proportionality.

Exhibit 3 also represents this model, but the nature of \( a_{ij} \) (and also \( d_{ij} \)) is now represented by the lower graph of Exhibit 2, instead of the upper graph of Exhibit 1. The cost of this model can be obtained from the cost of the “simple” recursive model by modifying the costs of determining matrices A and D to include the effects of the time measurements and observations needed to establish the linear
functions and the distributions of $a_{ij}$ and $d_{ij}$. Using an approach similar to the ones used above, the result is:

$$\text{TOTAL MODEL COST} = (n^2C_a)p + (n^2C_d)p + n(k + 1)C_b \quad (34)$$
$$= n(2npm + k + 1)C \text{ if } C_a = C_d = C_b = C \quad (35)$$

Note that (35) reduces to (28), as it should, if $p = m = 1$.

6. Cost Comparisons

We set out to show that the open-loop static model could be modified and converted to a recursive model suitable for forecasting. We accomplished this by developing a number of extension models which related the output at time $t + 1$ to the output at time $t$, a requirement of the forecasting model. But, it became apparent, as we were developing these models, that the cost requirements of the extension models also changed substantially from the cost of the static model. In this section we compare the costs of the extension models to the cost of the static model by using relative cost as the criterion of comparison where:

$$\text{Relative Cost} = \frac{\text{Cost of Extension Model}}{\text{Cost of Static Model}} \quad (36)$$

The results of this comparison are shown in Exhibit 4 below, which also summarizes the cost functions of the models:

**Exhibit 4 – Model Cost Comparisons**

<table>
<thead>
<tr>
<th>Model and Factors</th>
<th>Model Cost Function</th>
<th>Relative Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ___ ___ ___</td>
<td>$n(n + 1)C$</td>
<td>$\frac{n + 1}{np + 1} = p$</td>
</tr>
<tr>
<td>2) ___ ___ T</td>
<td>$n(np + 1)C$</td>
<td>$\frac{n + 1}{nm + 1} = m$</td>
</tr>
<tr>
<td>3) ___ R ___</td>
<td>$n(nm + 1)C$</td>
<td>$\frac{n + 1}{nmp + 1} = mp$</td>
</tr>
<tr>
<td>4) ___ R ___ T</td>
<td>$n(nmp + 1)C$</td>
<td>$\approx 2 + \frac{k}{n}$</td>
</tr>
<tr>
<td>5) S ___ ___</td>
<td>$n(2n + k + 1)C$</td>
<td>$\approx 2 + \frac{k}{n}$</td>
</tr>
<tr>
<td>6) S ___ T</td>
<td>$n(2np + k + 1)C$</td>
<td>$\approx 2p + \frac{k}{n}$</td>
</tr>
<tr>
<td>7) S R ___</td>
<td>$n(2nm + k + 1)C$</td>
<td>$\approx 2m + \frac{k}{n}$</td>
</tr>
<tr>
<td>8) S R T</td>
<td>$n(2nmp + k + 1)C$</td>
<td>$\approx 2mp + \frac{k}{n}$</td>
</tr>
</tbody>
</table>
The “complete” recursive model has all of the characteristics of the “simple” recursive model except that it treats the \(a_{ij}\) and \(d_{ij}\) coefficients as both functions of time and random variables. It is suitable for forecasting... but the relative cost of this model is proportionately higher...

where:

\[p: \text{number of measurements, over time, on } a_{ij} \text{ and } d_{ij}\]
\[m: \text{number of data needed to establish distributions}\]
\[k: \text{number of periods for which } b_t \text{ is defined}\]
\[n: \text{number of interacting economy segments}\]
\[C: C_a = C_d = C_b = C\]

As expected, the relative cost of the extension models is proportional to the number of measurements over time and observations needed to establish the \(a_{ij}\) and \(d_{ij}\) characteristics.

If \(p\), \(m\), and \(k\) are large, the relative cost of the corresponding extension model will also be large. It remains to be seen, from the forecasting capability of the model, whether such an added cost can be justified or not.

7. Conclusions

When some or all of the model modifications are incorporated into the basic (open-loop) static model they will render the model less rigid and more meaningful. But, to incorporate even one of these modifications, additional data will be needed to estimate the additional parameters introduced in the model by these modifications. The amount of additional data required depends on the complexity of the model chosen. Since additional data implies additional cost, the additional model cost can be justified only if the forecasting capability of the model can be established. This is another aspect of the problem not covered in this paper. But we can list the following conclusions:

1. The static (open-loop) model, even though it is very rigid, is good for studying the interactions among the segments of an economy, but only at one point in time. The model cannot be used for forecasting purposes.

2. Some of the rigidity of the model can be removed by making the \(a_{ij}\) coefficients functions of time and random variables. But still no real forecasting is possible (simple extrapolation is possible if time functions can also be obtained for the exogenous demands). However, the interactions among the segments of the economy can be studied at several points in time and the results can be considered more realistic since the \(a_{ij}\) coefficients are modeled more accurately, since they are selected from an entire distribution rather than being represented by a single value. The relative cost of the model is proportional to the product \(mp\).

3. The “simple” recursive model converts the open loop (static) model to a closed-loop model, suitable for forecasting, by introducing the concept of “stocks of capital goods,” but the model still treats the \(a_{ij}\)
and $d_i$ coefficients as fixed constants of proportionality, an assumption which is not very realistic. The relative cost of this model is approximately equal to $2 + (k/n)$.

4. The “complete” recursive model has all of the characteristics of the “simple” recursive model except that it treats the $a_{ij}$ and $d_{ij}$ coefficients as both functions of time and random variables. It is suitable for forecasting and the results are expected to be more “realistic” since the $a_{ij}$ and $d_{ij}$ coefficients are modeled more accurately. But the relative cost of this model is proportionately higher and is given by $2mp + (k/n)$, when $n >> 1$.

5. The relative cost of the “complete” recursive model can be substantially higher but, if it can be shown that this model produces “reasonable” forecasts, the higher cost may be justifiable.

References


The Role of Modeling in Scientific Disciplines: A Taxonomy

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Abstract
Models are ubiquitous across disciplines, but the model as a means of conveying knowledge and abstracting reality is little studied. Scholars in different disciplines may use a variety of divergent types of models and each understands his or her world through a prismatic view provided by the specialized model used. If each field focuses only on its own variety of model and each model has shortcomings, we’re still missing the big picture. The authors propose a taxonomy that may be used to classify the universe of models as used in numerous scientific and non-scientific disciplines. Such a framework can enable scholars to better communicate with their counterparts in other disciplines and to adapt the models of other disciplines to their own, thereby gaining a better understanding of their own specialized areas of study.

Introduction
The past decade has witnessed a convergence of technology more fantastic than science fiction. This is primarily due to widespread digitization and to the Internet, which can be seen as digitization plus telephony. Some of the results of this trend are: telephone companies provide cable, and cable companies provide telephone service; companies that produce printers are now in the camera business; long distance telephone calls use a broadband internet connection; photographs are transmitted via e-mail using a cellular telephone; several companies are competing for video-on-demand; computer manufacturers are in the music business; and many more. Convergence is even evidenced on the user side, in consumption convergence [6], meaning that consumers may be using several media simultaneously, e.g., computer, internet, music, newspapers, telephone, camera, etc., and in consumers who produce “mash-ups” using several forms of media.

Many of the most important discoveries of this century were made possible by individuals whose interests and training extended beyond narrow disciplinary boundaries.

The purpose of this paper is to investigate the use of models across disciplines, to try to create a framework in which diverse areas of scientific endeavor are represented specifically due to that discipline’s use of a model. The authors feel that such a framework will go a long way towards providing a structure for further interdisciplinary scholarly work, a way for scholars from different areas of study...
to establish common language and thereby develop new paradigms for research in this age of disciplinary convergence.

Abstraction models reality or, at the very least, a chosen view of reality in which irrelevant objects or properties are ignored ... making the model simpler conceptually and easier to study, manipulate and implement.

Convergence of Disciplines

Just as technological innovations are converging across product categories, and as product distinctions and corporate mission statements have become rapidly dated, so too scientific disciplines have been converging as well. Indeed, many of the most important discoveries of this century were made possible by individuals whose interests and training extended beyond narrow disciplinary boundaries. It is interesting to note, for example, that the technology for developing the laser existed for several decades before the first working laser was demonstrated in 1960. Part of the technology needed for developing lasers was well known in the electrical engineering field, that of oscillating amplifiers used in radio (light amplification). The quantum mechanical aspects of laser technology were well known in atomic physics, and were developed in the 1920’s by Albert Einstein (stimulated emission of radiation). The laser could not, however, be perfected until individuals who knew the two fields could combine the two technologies. Much technology works this way and it is therefore, in many cases, important for individuals to have knowledge of two or more fields. Increasingly, we find that the distinctions between individual disciplines are blurring and are due more to historical traditions than to substantive differences.

Johnson-Laird [7] points out that sometimes the same concept or methodology is examined by researchers from many diverse disciplines. For instance, decision making (logicians, statisticians, economists, psychologists) and parsing (mathematical linguists, psychologists, computer scientists). The past decade — even two decades — has seen a plethora of new inter- and multi-disciplinary studies [10]. What are the implications for education? Kolodny [11, pp. 40-41] asserts that interdisciplinary programs are crucial for students educated in the 21st century, and that the antiquated way of organizing colleges — by departments — will have to “evolve into collaborative and flexible units.” Perkins [13] feels that academic subjects are “artificial partitions with historic roots of limited contemporary significance.” Duderstadt [4] suggests that the university of the future will be very different from today’s institution. One major change will be that the future university will be divisionless, i.e., there will be many more interdisciplinary programs. There will also be “a far more intimate relationship between basic academic disciplines and the professions.” Duderstadt [3] asks us to consider “whether the concept of the disciplinary specialist is relevant to a future in which the most interesting and significant problems will require ‘big think’ rather than ‘small think.’ Indeed, the very survival of the university itself may require that it become a ‘learning organization.’ The learning organization is characterized, among other traits, by its flexibility and free flow of knowledge. This is expressed, in the context of a university, as significant cross-disciplinary synergies both inside and outside the school [16].

Students seem to have come up with their own solution: majoring in one, two, three, four, or even five fields. Lewin [12] notes that the number of multiple majors is increasing at most universities. He cites the following interesting statistics: At Georgetown University, the number of double majors grew from 14% of the class in 1996 to 23% in 2002; at Washington
The Role of Modeling in Scientific Disciplines: A Taxonomy

University, the number of double majors rose from 28% in 1997 to 42% in 2002. Weber [14] states that about 25% to 35% of students at major graduate schools of business are now pursuing double majors (e.g., JD/MBA, MD/MBA, MPP/MPA, ME/MBA, and MSW/MBA to name a few). The purpose is to make oneself more marketable in a difficult job market.

The Role of Models

With the convergence of disciplines it becomes important to look at the way we handle information — studying, analyzing, managing, reducing, employing, etc. — in various diverse areas of study. How do scholars wrap their minds around vast quantities of information in diverse formats? Is it even possible? If we can find common ground in the way information is handled in the process of acquiring knowledge in disparate fields of human endeavor, perhaps it will be a first step towards this goal.

Every field of human endeavor requires us to process data into information. In any modeling effort, abstraction and structure are important universal features. These two related concepts are involved in the reduction and management of complexity. Perhaps the best or, at least, the most parsimonious, definition of model is that it is a representation of a real-world entity but not the “real thing” itself. This definition, necessarily vague, encompasses just about any type of model in the spectrum, from physical models, like the full-scale mock-ups used to train airplane pilots, to the heuristic models of today’s expert systems. Abstraction models reality or, at the very least, a chosen view of reality in which irrelevant objects or properties are ignored in favor of streamlining the model, thus making the model simpler conceptually and easier to study, manipulate, and implement.

The general model governing abstraction is the so-called black box model [e.g. 1], adopted from the engineering disciplines to many diverse areas. In this model, a set of inputs is mapped to a set of outputs or results by means of a transform. To use the transform, once it has been built, one need not know how it works; only that it does work. For example, we do not need to understand much about electricity to know that when we flip the light switch (input), the bulb will light up (output).

Much like the parable of the “Blind Men and the Elephant,” scholars in different disciplines may use a variety of divergent types of models, and each understands his or her world through a prismatic view provided by the specialized model used.

All abstraction uses the concept of information hiding. When models are well designed, they are relatively independent. They communicate with each other only through well defined interfaces. A “user” system does not require access to all the implementation details of the “used.” This unnecessary information may be hidden from the user, protecting the integrity of individual systems and reducing the confusion that comes along with too much information.

Abstraction also allows one to ignore the tedious details (at least temporarily) involved in building a system, and concentrate on the larger picture. Abstraction is the major concept used in bottom up design: the construction of a large program by building layers upon layers of abstraction. Abstraction comes in many forms. Every field has its own specialized model, from the concrete to the abstract, from the real (say, a fashion model) to the simulated models of virtual reality.
Modeling refers to the various techniques we use to understand our world. The process of modeling includes analysis, abstraction, simplification, and approximation. Assumptions are made and tested. Simplicity, efficiency considerations, and the goals of the modeling effort are the prime determinants of the type of model to be employed; sometimes, a combination of modeling techniques is indicated.

Methodology

In an early attempt at creating a family tree of models in different disciplines, Friedman's [9] objective was to indicate the relative position of system simulation models — these are (usually) large, complex computer programs that represent a dynamic, probabilistic system composed of people, machinery, computers, processes, etc. — in the modeling “family tree” as indicated in Exhibit 1.

Exhibit 1
A View of the Modeling Family Tree

For the current study, the authors explored the models used in diverse disciplines and attempted to create a unifying framework with which to build a taxonomy, in order to provide not only a structure for further interdisciplinary and multidisciplinary scholarly work, but a way for scholars from different areas of study to establish common language for research in this age of disciplinary convergence.

Disciplines examined include: natural science, mathematics, literature, social sciences, theology, education, business, humanities, art, physics, chemistry and computer science, among others. Indeed, there seemed to be no field of human scholarship which did not employ the general concept of model to study the world using its own unique point of view.

Findings

Models are ubiquitous across disciplines, but the model itself is little studied. Rather, it is considered a tool with which to study a particular domain. Frantz [8] examined and classified modeling techniques used in simulation and artificial intelligence. Gemino and Wand [5] noted the equivalence between systems analysis modeling and such diverse techniques as scale models, blueprints and a movie storyboard. The model technique may change to fit the domain, but the ideas behind the modeling effort as well as the benefits of improved communication are still there. According to these authors, people learn from models using prior experience.

The mental model, a concept from the realm of cognitive science, is an “internal” model [2] that we use to understand reality and help us make decisions. Johnson-Laird [7] notes that in some disciplines, the main thing you want to make sure of is that theories correspond to facts; in others, it is more important to make sure that theories are coherent. He feels that cognitive science requires both.
Exhibit 2
A Taxonomy of Models

Exhibit 2 is our attempt to classify the universe of models as used in various scientific and non-scientific disciplines. The taxonomy is multi-level. At the first level, models are grouped into concrete (e.g., a physical scale model such as a model airplane), abstract (e.g., an analytical formula or numerical approximation) and mimetic (e.g., a work of art or a virtual reality environment). Then, within each broad category of model, the group is further classified.

Scholars working in one discipline, by understanding how models are used in other areas of study, will be able to develop new types of models and thus ultimately gain a better understanding of their own discipline.

Concrete, or physical, models can be two-dimensional, e.g., the blueprints used by architects, or three-dimensional, such as the architect’s miniaturized rendering of a large building complex, a model of an atom or a molecule, or a clothing model in a department store window. Abstract models can be analytical formulas of, e.g., statistics, queuing or econometrics; the numerical models used in simulation; or theoretical models such as those of quantum physics, chaos theory, the technology acceptance model (TAM) of information systems, or Kuhn’s paradigms of scientific research.

Mimetic models are those that involve imitation of life or the creation of alternative universes. They include man-machine models such as venture capital games — indeed, most gaming models — and virtual reality environments. Man-man models can include those used in psychology and education, for example: role playing, role models, and the what-if scenarios used in the military, law enforcement and intelligence communities. Also included in the mimetic models branch are the models of culture or society used in art, fiction, film and theater. These models may represent a version of the world as it really is or, often, an alternative version of the world as the author or artist imagines it to be, such as a utopian or dystopian view of the world.
Conclusion

Why are there so many different kinds of models? One reason is that individuals in the various disciplines are used to thinking in a certain way. Second, it may relate to the purpose of the model. The model may be trying to describe reality; or it may be a predictive model. Much like the parable of the “Blind Men and the Elephant,” scholars in different disciplines may use a variety of divergent types of models, and each understands his or her world through a prismatic view provided by the specialized model used. If each field focuses on its own variety of model, and each model has shortcomings, we are still missing the larger picture. When researchers in different disciplines collaborate (say, e.g., in bioinformatics), the first thing they have to do is understand each other’s models, if only to aid in communication.

A medical researcher probably could not make very good use of a portrait painted by an artist, even a life-sized marble statue of a human. Similarly, an artist might not be able to use a chemical formula to paint a portrait. However, both types of models give us a partial view into what it means to be human.

In the field of psychology, the treatment of phobias and anxiety disorders has relied mostly on “man-man” models. Recently, however, there has been some exciting research into the use of virtual reality environments for the treatment of these disorders [15]. This represents a crossing over from the “man-man” branch to the “man-machine” branch of mimetic models.

What do the models in Exhibit 2 have in common? Do they even belong on the same tree? An examination of the various models represented in the classification tree reveals that there is indeed some commonality in the way these very different fields process information. Each model is a view of reality; has a purpose; and employs abstraction, structure, and information hiding. In addition, each model alters reality to some degree. Even physical models, which we may expect to be fairly good representations, may be faster, slower, flatter, larger, or smaller than the reality they purport to represent.

Scholars working in one discipline, by understanding how models are used in other areas of study, will be able to develop new types of models and thus ultimately gain a better understanding of their own discipline. One excellent example of this is the case study. Case studies evolved during the 20th century out of the case histories used in medicine, as a teaching tool. Case studies are now used in such diverse disciplines as sociology, management, ethics, economics, and the military. As another example, experimental design, widely used in psychology, is now being used as an important tool in economics.

One thing is clear from studying this proposed classification scheme. There are many kinds of models, and each has its advantages. There is no reason for scholars in one discipline to limit themselves to only one kind of model. Certainly scholars who wish to cross interdisciplinary boundaries have to understand the models used by the disciplines on the other side. It is the authors’ hope that this paper might provide a means of communication for scholars engaged in interdisciplinary work. Indeed, even for scholars not engaged in research that crosses disciplinary boundaries, an understanding of the models used by others can only serve to enhance their own work.
References
